MHT-CET MATHEMATICS PAPER - 2022

11TH AUGUST (SHIFT - 2)

Time : 90 Minutes **No. of Questions : 50** **Marks : 100**

1. $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x}$ is equal to

(a) $-2\sqrt{2}$ **(b)** 2
(c) $-2 - 2\sqrt{2}$ **(d)** -2

2. If $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$, then x has the value

(a) 3 **(b)** 1
(c) $\frac{1}{\sqrt{3}}$ **(d)** $\sqrt{3}$

3. If $p : \forall\, n \in IN,\ n^2 + n$ is an even number
$q : \forall\, n \in IN,\ n^2 - n$ is an odd numer,
then the truth values of $p \wedge q$, $p \vee q$ and $p \to q$ are respectively

(a) F, T, F **(b)** F, F, T
(c) T, T, F **(d)** F, T, T

4. If the function $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$, for some $a \in IR$ is increasing in $(0, 1]$ and decreasing in $[1, 5)$, then a root of the equation $\frac{f(x) - 14}{(x-1)^2} = 0\ (x \neq 1)$ is

(a) 14 **(b)** 7
(c) -14 **(d)** -7

5. If $\bar{a} = \hat{i} - \hat{k}$, $\bar{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and $\bar{c} = y\hat{i} + x\hat{j} + (1 + x - y)\hat{k}$, then $[\bar{a}\ \bar{b}\ \bar{c}]$ depends on

(a) only y **(b)** neither x nor y
(c) both x and y **(d)** only x

6. If $\cot(A + B) = 0$, then $\sin(A + 2B)$ is equal to

(a) $\sin A$ **(b)** $\cos 2A$
(c) $\sin 2A$ **(d)** $\cos A$

7. The joint equation of pair of lines through the origin and making an equilateral triangle with the line $y = 5$ is

(a) $x^2 - 3y^2 = 0$ **(b)** $\sqrt{3}x^2 - y^2 = 0$
(c) $3x^2 - y^2 = 0$ **(d)** $5x^2 - y^2 = 0$

8. If $f(x) = \sqrt{\tan x}$ and $g(x) = \sin x \cdot \cos x$, then $\int \frac{f(x)}{g(x)}\, dx$ is equal to
(where C is a constant of integration)

(a) $2\sqrt{\tan x} + C$ **(b)** $\frac{1}{2}\sqrt{\tan x} + C$
(c) $\sqrt{\tan x} + C$ **(d)** $\frac{3}{2}\sqrt{\tan x} + C$

9. The general solution of the differential equation $\frac{dy}{dx} = \frac{3x + y}{x - y}$ is (where C is a constant of integration.)

(a) $\tan^{-1}\left(\frac{y}{x}\right) + \log\left(\frac{y^2 + 3x^2}{x^2}\right) = \log(x) + C$

(b) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{y}{x\sqrt{3}}\right) + \log\left(\frac{y^2 + 3x^2}{x^2}\right)^{\frac{1}{2}} = \log(x) + C$

(c) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{y}{x\sqrt{3}}\right) - \log\left(\frac{y^2 + 3x^2}{x^2}\right)^{\frac{1}{2}} = \log(x) + C$

(d) $\tan^{-1}\left(\frac{x}{y}\right) + \log\left(\frac{y^2 + 3x^2}{x^2}\right) = \log(x) + C$

10. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 2 \end{bmatrix}$, then $(AB)^{-1} =$

(a) $\begin{bmatrix} \frac{17}{5} & \frac{9}{5} \\ 2 & 1 \end{bmatrix}$ **(b)** $\begin{bmatrix} \frac{-17}{5} & \frac{9}{5} \\ 2 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} \frac{17}{5} & 2 \\ \frac{9}{5} & 1 \end{bmatrix}$ **(d)** $\begin{bmatrix} \frac{-17}{5} & 2 \\ \frac{-9}{5} & -1 \end{bmatrix}$

11. The distance between parallel lines

$\frac{x-1}{2} = \frac{y-2}{-2} = \frac{z-3}{1}$ and $\frac{x}{2} = \frac{y}{-2} = \frac{z}{1}$ is

(a) $\frac{2\sqrt{5}}{3}$ units **(b)** $\frac{\sqrt{5}}{3}$ units

(c) $\frac{5\sqrt{5}}{3}$ units **(d)** $\frac{4\sqrt{5}}{3}$ units

12. Maximum value of $Z = 5x + 2y$,
subject to $2x - y \geq 2,\ x + 2y \leq 8$ and $x, y \geq 0$ is

(a) 40 **(b)** 17.6

(c) 28 **(d)** 25.6

13. The value of $\sin(2\sin^{-1}0.8)$ is equal to

(a) 0.96 **(b)** 0.16

(c) 0.12 **(d)** 0.48

14. A line makes the same angle 'α' with each of the x and y axes. If the angle 'θ', which it makes with the z-axis, is such that $\sin^2\theta = 2\sin^2\alpha$, then the angle α is

(a) $\left(\frac{\pi}{4}\right)$ **(b)** $\left(\frac{\pi}{2}\right)$

(c) $\left(\frac{\pi}{3}\right)$ **(d)** $\left(\frac{\pi}{6}\right)$

15. The negation of the statement pattern $p \vee (q \rightarrow \sim r)$ is

(a) $\sim p \wedge (q \wedge \sim r)$ **(b)** $\sim p \wedge (q \wedge r)$

(c) $\sim p \wedge (\sim q \wedge r)$ **(d)** $\sim p \wedge (\sim q \wedge \sim r)$

16. Let $\bar{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\bar{b} = \hat{i} - \hat{j} + \hat{k}$ be two vectors. If $\bar{c}$ is a vector such that $\bar{b} \times \bar{c} = \bar{b} \times \bar{c}$ and $\bar{c} \cdot \bar{a} = 0$, then $\bar{c} \cdot \bar{b}$ is equal to

(a) $-\frac{1}{2}$ **(b)** $\frac{3}{2}$

(c) $\frac{1}{2}$ **(d)** $-\frac{3}{2}$

17. The incidence of occupational disease in an industry is such that the workmen have a 10% chance of suffering from it. The probability that out of 5 workmen, 3 or more will contract the disease is

(a) 0.000856 **(b)** 0.856

(c) 0.0000856 **(d)** 0.00856

18. If $y = \log\sqrt{\frac{1+\sin x}{1-\sin x}}$, then $\frac{dy}{dx}$ at $x = \frac{\pi}{3}$ is

(a) $\frac{1}{2}$ **(b)** $-\frac{1}{2}$

(c) 2 **(d)** $\frac{1}{4}$

19. The variance and mean of 15 observations are respectively 6 and 10. If each observation is increased by 8 then the new variance and new mean of resulting observations are respectively

(a) 6, 18 **(b)** 6, 10

(c) 14, 10 **(d)** 14, 18

20. If $y = \sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{-x}{\sqrt{1-x^2}}$ **(b)** $\frac{-2x}{\sqrt{1-x^2}}$

(c) $\frac{-1}{\sqrt{1-x^2}}$ **(d)** $\frac{1}{\sqrt{1-x^2}}$

21. The magnitude of the projection of the vector $2\hat{i} + 3\hat{j} + \hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$ is

(a) $3\sqrt{6}$ units **(b)** $\frac{\sqrt{3}}{2}$ units

(c) $\frac{3}{\sqrt{2}}$ units **(d)** $\sqrt{\frac{3}{2}}$ units

22. If $f(x) = ax^2 + bx + 1$, if $|2x - 3| \geq 2$

$= 3x + 2$, if $\frac{1}{2} < x < \frac{5}{2}$

is continuous on its domain, then $a + b$ has the value

(a) $\frac{13}{5}$ **(b)** $\frac{31}{5}$

(c) $\frac{23}{5}$ **(d)** $\frac{1}{5}$

23. If $\bar{a} = \hat{i} + \hat{j} + \hat{k}$, $\bar{b} = \hat{i} - \hat{j} + \hat{k}$ and $\bar{c} = \hat{i} - \hat{j} - \hat{k}$ are three vectors then vector $\bar{r}$ in the plane of $\bar{a}$ and $\bar{b}$, whose projection on $\bar{c}$ is $\frac{1}{\sqrt{3}}$, is given by

(a) $(2t+1)\hat{i} - \hat{j} + (2t+1)\hat{k},\ \forall t \in IR$

(b) $(2t+1)\hat{i} - \hat{j} + (2t-1)\hat{k},\ \forall t \in IR$

(c) $(2t-1)\hat{i} - \hat{j} + (2t-1)\hat{k},\ \forall t \in IR$

(d) $(2t-1)\hat{i} - \hat{j} + (2t+1)\hat{k},\ \forall t \in IR$

24. A tetrahedron has verticles $P(1, 2, 1)$, $Q(2, 1, 3)$, $R(-1, 1, 2)$ and $O(0, 0, 0)$. Then the angle between the faces OPQ and PQR is

(a) $\cos^{-1}\left(\frac{17}{35}\right)$ **(b)** $\cos^{-1}\left(\frac{17}{31}\right)$

(c) $\cos^{-1}\left(\frac{19}{35}\right)$ **(d)** $\cos^{-1}\left(\frac{19}{31}\right)$

25. The principal value of $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$ is

(a) $\left(\frac{\pi}{3}\right)$ **(b)** $\left(\frac{2\pi}{3}\right)$

(c) $-\left(\frac{2\pi}{3}\right)$ **(d)** $\left(\frac{5\pi}{3}\right)$

26. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with

(a) fixed radius of 1 unit and variable centres along the X-axis

(b) fixed radius of 1 unit and variable centres along the Y-axis

(c) variable radii and a fixed centre at $(0, 1)$

(d) variable radii and a fixed centre at $(0, -1)$

27. The value of the integral $\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx$ is

(a) $\left(\frac{\pi}{2}\right) + 1$ **(b)** $\left(\frac{\pi}{2}\right) - 1$

(c) 1 **(d)** -1

28. If a question paper consists of 11 questions divided into two sections I and II. Section I consists of 6 questions and section II consists of 5 questions, then the number of different ways can student select 6 questions, taking at least 2 questions from each section, is

(a) 425 **(b)** 275

(c) 350 **(d)** 225

29. The area (in sq. units) of the region described by $A = \{(x, y)/x^2 + y^2 \le 1 \text{ and } y^2 \le 1 - x\}$ is

(a) $\left(\frac{\pi}{2} + \frac{2}{3}\right)$ **(b)** $\left(\frac{\pi}{2} + \frac{4}{3}\right)$

(c) $\left(\frac{\pi}{2} - \frac{4}{3}\right)$ **(d)** $\left(\frac{\pi}{2} - \frac{2}{3}\right)$

30. A firm is manufacturing 2000 items. It is estimated that the rate of change of production P with respect to additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is

(a) 4500 **(b)** 3000

(c) 2500 **(d)** 3500

31. $\int \frac{3x-2}{(x+1)(x-2)^2}\,dx =$

(where C is a constant of integration)

(a) $\frac{-5}{9}\log(x+1) + \frac{5}{9}\log(x-2) - \frac{4}{3} \times \frac{1}{(x-2)} + C$

(b) $\frac{-5}{9}\log(x+1) + \frac{5}{9}\log(x-2) - \frac{1}{x-2} + C$

(c) $\frac{1}{9}\log(x+1) + \frac{5}{9}\log(x-2) - \frac{4}{3} \times \frac{1}{(x-2)} + C$

(d) $\frac{-5}{9}\log(x+1) + \frac{1}{9}\log(x-2) - \frac{1}{x-2} + C$

32. If the normal to the curve $y = f(x)$ at the point $(3, 4)$ makes an angle $\left(\frac{3\pi}{4}\right)^c$ with positive X-axis, then $f'(3)$ is equal to

(a) -1 **(b)** $\frac{4}{3}$

(c) $-\frac{3}{4}$ **(d)** 1

33. If $P(A \cup B) = 0.7$, $P(A \cap B) = 0.2$, then $P(A') + P(B')$ is

(a) 1.1 **(b)** 1.6

(c) 1.8 **(d)** 0.6

34. If $\lim\limits_{x \to 1} \dfrac{x^2 - ax + b}{(x-1)} = 5$, then $(a + b)$ is equal to

(a) -4 **(b)** -7
(c) 7 **(d)** -3

35. If $y = \cos(\sin x^2)$, then $\dfrac{dy}{dx}$ at $x = \sqrt{\dfrac{\pi}{2}}$ is

(a) 0 **(b)** 2
(c) -1 **(d)** -2

36. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Then mean of number of kings is

(a) $\dfrac{4}{169}$ **(b)** $\dfrac{1}{13}$
(c) $\dfrac{1}{169}$ **(d)** $\dfrac{2}{13}$

37. The polar co-ordinates of the point, whose Cartesian co-ordinates are $(-2\sqrt{3}, 2)$, are

(a) $\left(4, \left(\dfrac{3\pi}{4}\right)\right)$ **(b)** $\left(4, \left(\dfrac{5\pi}{6}\right)\right)$
(c) $\left(4, \left(\dfrac{2\pi}{3}\right)\right)$ **(d)** $\left(4, \left(\dfrac{11\pi}{12}\right)\right)$

38. Let z be a complex number such that $|z| + z = 3 + i$, $i = \sqrt{-1}$, then $|z|$ is equal to

(a) $\dfrac{5}{4}$ **(b)** $\dfrac{5}{3}$
(c) $\dfrac{\sqrt{34}}{3}$ **(d)** $\dfrac{\sqrt{41}}{4}$

39. Given $A = \begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$, if $xyz = 60$ and $8x + 4y + 3z = 20$, then A

(a) $\begin{bmatrix} 60 & 0 & 0 \\ 0 & 60 & 0 \\ 0 & 0 & 60 \end{bmatrix}$ **(b)** $\begin{bmatrix} 108 & 0 & 0 \\ 0 & 108 & 0 \\ 0 & 0 & 108 \end{bmatrix}$
(c) $\begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$ **(d)** $\begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$

40. If $y^{\frac{1}{m}} + y^{\frac{-1}{m}} = 2x$, $x \neq 1$, then $(x^2 - 1)\left(\dfrac{dy}{dx}\right)^2$ is equal to

(a) my^2 **(b)** m^2y
(c) m^2y^2 **(d)** $\dfrac{my^2}{2}$

41. $\int_0^2 [x]\,dx + \int_0^2 |x - 1|\,dx =$
(where $[x]$ denotes the greatest integer function.)

(a) 4 **(b)** 3
(c) 1 **(d)** 2

42. The equations of the lines passing through the point $(3, 2)$ and making an acute angle of 45° with the line $x - 2y - 3 = 0$ are

(a) $3x + y - 11 = 0,\ x + 3y + 9 = 0$
(b) $3x - y - 7 = 0,\ x + 3y - 9 = 0$
(c) $3x + y - 11 = 0,\ x + 3y - 9 = 0$
(d) $x + 2y - 7 = 0,\ 2x - y - 4 = 0$

43. If $[x]$ is greatest integer function and $2[2x - 5] - 1 = 7$, then x lies in

(a) $\left(\dfrac{9}{2}, 5\right)$ **(b)** $\left(\dfrac{9}{2}, 5\right]$
(c) $\left[\dfrac{9}{2}, 5\right]$ **(d)** $\left[\dfrac{9}{2}, 5\right)$

44. The Cartesian equation of a line passing through $(1, 2, 3)$ and parallel to $x - y + 2z = 5$ and $3x + y + z = 6$ is

(a) $\dfrac{x-1}{-3} = \dfrac{y-2}{5} = \dfrac{z-3}{4}$
(b) $\dfrac{x-1}{3} = \dfrac{y-2}{1} = \dfrac{z-3}{1}$
(c) $\dfrac{x-1}{1} = \dfrac{y-2}{-1} = \dfrac{z-3}{1}$
(d) $\dfrac{x-1}{-3} = \dfrac{y-2}{-5} = \dfrac{z-3}{4}$

45. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ pass through diameters of a circle of area 49π square units, then the equation of the circle is

(a) $x^2 + y^2 - 2x + 2y - 47 = 0$
(b) $x^2 + y^2 - 2x + 2y + 51 = 0$
(c) $x^2 + y^2 + 2x - 2y - 51 = 0$
(d) $x^2 + y^2 + 2x + 2y + 47 = 0$

46. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness that melts at the rate of 50 cm^3/min. If the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases is

(a) $\frac{-1}{18\pi}$ cm/min **(b)** $\frac{2}{9\pi}$ cm/min

(c) $\frac{1}{18\pi}$ cm/min **(d)** $\frac{1}{3\pi}$ cm/min

47. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}\,dx =$

(where C is a constant of integration.)

(a) $x + \sin x + \sin 2x + C$

(b) $x + \sin x + \sin 2x - C$

(c) $x + 2\sin x + 2\sin 2x + C$

(d) None of these

48. The equation of the plane passing through the points $(2, 3, 1), (4, -5, 3)$ and parallel to X-axis is

(a) $3y + 4z = 13$ **(b)** $y - 4z = -1$

(c) $2y + 4z = 19$ **(d)** $y + 4z = 7$

49. If $\int e^{x^2} \cdot x^3\,dx = e^{x^2} \cdot [f(x) + C]$

(where C is a constant of integration.)

and $f(1) = 0$, then value of $f(2)$ will be

(a) $\frac{-3}{2}$ **(b)** $\frac{-1}{2}$

(c) $\frac{3}{2}$ **(d)** $\frac{1}{2}$

50. The negation of the statement, "The payment will be made if and only if the work is finished in time" is

(a) The work is finished in time and the payment is not made or the payment is made and the work is finished in time.

(b) The work is finished in time and the payment is not made.

(c) The payment is made and the work is not finished in time.

(d) Either the work is finished in time and the payment is not made or the payment is made and the work is not finished in time.

ANSWERS

1. b	**2.** c	**3.** a	**4.** b	**5.** b
6. a	**7.** b	**8.** a	**9.** c	**10.** b
11. c	**12.** a	**13.** a	**14.** a	**15.** b
16. c	**17.** d	**18.** c	**19.** d	**20.** a
21. d	**22.** a	**23.** a	**24.** a	**25.** b
26. a	**27.** b	**28.** b	**29.** a	**30.** d
31. a	**32.** d	**33.** a	**34.** b	**35.** a
36. d	**37.** b	**38.** c	**39.** c	**40.** c
41. d	**42.** b	**43.** c	**44.** d	**45.** a
46. c	**47.** d	**48.** d	**49.** c	**50.** d

HINTS / SHORT-CUTS / SOLUTIONS

1. $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x} = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{2\cdot\cos^2\frac{x}{2}}$

$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left(\frac{1}{2}\sec^2\frac{x}{2}\right)dx = \left[\tan\frac{x}{2}\right]_{\pi/4}^{3\pi/4}$

$= \tan\frac{3\pi}{8} - \tan\frac{\pi}{8} = \frac{\sin(3\pi/8)}{\sin(3\pi/8)} - \frac{\sin(\pi/8)}{\cos(\pi/8)}$

$= \frac{\sin(3\pi/8)\cdot\cos(\pi/8) - \cos(3\pi/8)\cdot\sin(\pi/8)}{\cos(3\pi/8)\cdot\cos(\pi/8)}$

$= \frac{\sin\left(\frac{3\pi}{8} - \frac{\pi}{8}\right)}{\frac{1}{2}\left[\cos\left(\frac{3\pi}{8} + \frac{\pi}{8}\right) + \cos\left(\frac{3\pi}{8} - \frac{\pi}{8}\right)\right]}$

$= \frac{2\times\sin\frac{\pi}{4}}{\cos\frac{\pi}{2} + \cos\frac{\pi}{4}} = \frac{2\times\frac{1}{\sqrt{2}}}{0 + \frac{1}{\sqrt{2}}} = 2$ **... (b)**

2. $\because \tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\cdot\tan^{-1}x$

$\therefore 2\left[\tan^{-1}\left(\frac{1-x}{1+1\cdot x}\right)\right] = \tan^{-1}x$

$\therefore 2[\tan^{-1}1 - \tan^{-1}x] = \tan^{-1}x$

$\therefore 2\left(\frac{\pi}{4} - \tan^{-1}x\right) = \tan^{-1}x$

$\therefore\ \dfrac{\pi}{2} = 3\cdot\tan^{-1}x$

$\therefore\ \dfrac{\pi}{6} = \tan^{-1}x$

$\therefore\ x = \tan\dfrac{\pi}{6} = \dfrac{1}{\sqrt{3}}$ **... (c)**

3. $\forall\, n \in N,\ n^2 + n$ is an even number

but $n^2 - n$ is not an even number

$\therefore p$ is T and q is F

$\therefore p \wedge q = T \wedge F = \boldsymbol{F}$

$p \vee q = T \vee F = \boldsymbol{T}$

$p \to q = T \to F = \boldsymbol{F}$ **... (a)**

4. $\because f$ is $\uparrow$ in $(0, 1]$ and $\downarrow$ in $[1, 5)$

$\therefore f$ is both $\uparrow$ and $\uparrow$ at $x = 1$

$\therefore f(1) = \text{constant} \quad \therefore f'(1) = 0$

$\because f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$

$\therefore f'(x) = 3x^2 - 6(a-2)x + 3a$

$\therefore f'(1) = 3 - 6(a-2) + 3a = 15 - 3a = 0$

$\therefore a = 5$

$\therefore f(x) = x^3 - 9x^2 + 15x + 7$

$\therefore f(x) - 14 = x^3 - 9x^2 + 15x - 7$

$= (x-7)(x-1)^2$ ... factorising

$\therefore\ \dfrac{f(x) - 14}{(x-1)^2} = x - 7 = 0 \quad \therefore \boldsymbol{x = 7}$ **... (b)**

5. $\bar{a} = \hat{\boldsymbol{i}} - \hat{\boldsymbol{k}} \equiv (1, 0, -1)$

$\bar{b} = x\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + (1-x)\hat{\boldsymbol{k}} \equiv (x, 1, 1-x)$

$\bar{c} = y\hat{\boldsymbol{i}} + x\hat{\boldsymbol{j}} + (1+x-y)\hat{\boldsymbol{k}} \equiv (y, x, 1+x-y)$

$$\therefore [\bar{a}\ \bar{b}\ \bar{c}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ y & x & 1+x \end{vmatrix} \quad \text{... by } C_1 + C_3$$

$= 1\,[(1)(1+x) - (x)(1)]$

$= 1 + x - x$

$= 1$

$=$ contains neither x nor y **... (b)**

6. $\because \cot(A+B) = 0 \quad \therefore A + B = \dfrac{\pi}{2}$

$\therefore \sin(A+2B) = \sin\left[A + 2\left(\dfrac{\pi}{2} - A\right)\right]$

$= \sin(A + \pi - 2A)$

$= \sin(\pi - A)$

$= \sin A$ **... (a)**

7. Joint equation of a pair of lines, through the origin, making an equilateral triangle with the line $y = b$, is

$\sqrt{3}x^2 - y^2 = 0$ **... (b)**

8. $I = \displaystyle\int \frac{\sqrt{\tan x}}{\sin x\cdot\cos x}\,dx$

$= \displaystyle\int \frac{\sqrt{\tan x}}{\left(\dfrac{\sin x}{\cos x}\right)\cdot\cos^2 x}\,dx$

$= \displaystyle\int \frac{\sqrt{\tan x}}{\tan x}\cdot\frac{1}{\cos^2 x}\,dx$

$= \displaystyle\int \frac{1}{\sqrt{\tan x}}\cdot\sec^2 x\,dx$

$= \displaystyle\int \frac{1}{\sqrt{t}}\,dt$, ... $t = \tan x$

$= 2\cdot\sqrt{t} + C$

$= 2\cdot\sqrt{\tan x} + C$ **... (a)**

9. $\because \dfrac{dy}{dx} = \dfrac{3x+y}{x-y} \quad \therefore$ put $y = vx$

$\therefore v + x\cdot\dfrac{dv}{dx} = \dfrac{3+v}{1-v} \quad \therefore x\cdot\dfrac{dv}{dx} = \dfrac{3+v}{1-v} - v$

$\therefore x\cdot\dfrac{dv}{dx} = \dfrac{3+v-v+v^2}{1-v} \quad \therefore x\cdot\dfrac{dv}{dx} = \dfrac{3+v^2}{1-v}$

$\therefore \displaystyle\int \frac{1-v}{3+v^2}\,dv = \int \frac{1}{x}\,dv$

$\therefore \displaystyle\int \frac{1}{\sqrt{3}^2 + v^2}\,dv - \frac{1}{2}\int \frac{2v}{3+v^2}\,dv = \log x$

$\therefore \dfrac{1}{\sqrt{3}}\cdot\tan^{-1}\left(\dfrac{v}{\sqrt{3}}\right) - \dfrac{1}{2}\cdot\log(3+v^2) = \log x + C$

$\therefore \dfrac{1}{\sqrt{3}}\cdot\tan^{-1}\left(\dfrac{y}{x\sqrt{3}}\right) - \log\left(\dfrac{y^2+3x^2}{x^2}\right)^{1/2}$

$= \log x + C$ **... (c)**

10. **Note :** $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$A\cdot B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 3 \end{bmatrix}\cdot\begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 9 \\ 10 & 17 \end{bmatrix}$

$\therefore |AB| = 85 - 90 = -5$

$\therefore (AB)^{-1} = \dfrac{1}{-5}\begin{bmatrix} 17 & -9 \\ -10 & 5 \end{bmatrix} = \begin{bmatrix} -17/5 & 9/5 \\ 2 & -1 \end{bmatrix}$ **... (b)**

11. $L_1 : \dfrac{x-1}{2} = \dfrac{y-2}{-2} = \dfrac{z-3}{1}$

$L_2 = \dfrac{x-0}{2} = \dfrac{y-0}{-2} = \dfrac{z-0}{1}$

$\therefore$ their vector equations are

$\bar{r} = \bar{a}_1 + m\bar{b}$ and $\bar{r} = \bar{a}_2 + n\bar{b}$, where

$\bar{a}_1 \equiv (1, 2, 3)$, $\bar{a}_2 \equiv (0, 0, 0)$, $\bar{b} \equiv (2, -2, 1)$

$$\therefore (\bar{a}_1 - \bar{a}_2) \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & -3 \\ 2 & -2 & 1 \end{vmatrix} \equiv (-8, -5, 6)$$

$$\therefore |(\bar{a}_2 - \bar{a}_1) \times \bar{b}| = \sqrt{64 + 25 + 36} = 5\sqrt{5}$$

$$|\bar{b}| = \sqrt{4 + 4 + 1} = 3$$

$$\therefore d(L_1, L_2) = \left| \frac{(\bar{a}_2 - \bar{a}_1) \times \bar{b}}{b} \right| = \frac{5\sqrt{5}}{3}$$ **... (c)**

12. $Z = 5x + 2y$ **... (1)**

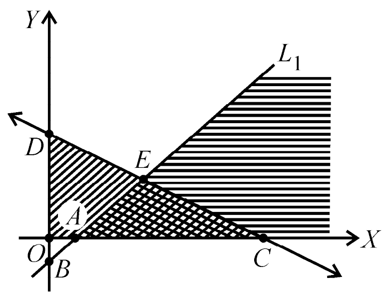
$2x - y \geq 2,\ x + 2y \leq 8,\ x, y \geq 0$

$L_1 : 2x - y = 2,\ L_2 : x + 2y = 8$

$\therefore A(1, 0),\ B(0, -2)$ lie on L_1

$C(8, 0),\ D(0, 4)$ lie on L_2

$$L_1 \cap L_2 \equiv \left(\frac{12}{5}, \frac{14}{5}\right) \equiv E$$



$\therefore$ Critical points are $A(1, 0)$, $C(8, 0)$, $E \equiv \left(\frac{12}{5}, \frac{14}{5}\right)$

$\therefore Z_A = 5(1) + 2(0) = 5$

$Z_C = 5(8) + 2(0) = 40$

$$Z_E = 5\left(\frac{12}{5}\right) + 2\left(\frac{14}{5}\right) = \frac{88}{5} = 17.6$$

$\therefore Z_{max} = \mathbf{40}$ **... (a)**

13. If $x = 0.8$, then

$$\sin(2\sin^{-1}x) = 2x \cdot \sqrt{1 - x^2}$$
$$= 2(0.8)\sqrt{1 - (0.8)^2}$$
$$= (1.6)\sqrt{1 - 0.64}$$
$$= (1.6)(0.6)$$
$$= 0.96$$ **... (a)**

14. $\because \cos^2\alpha + \cos^2\alpha + \cos^2\theta = 1$

$\therefore 1 - \cos^2\theta = 2\cos^2\alpha$

$\therefore \mathbf{\sin^2\theta} = 2\cos^2\alpha$

$\therefore 2\,\mathbf{\sin^2\alpha} = 2\cos^2\alpha$ ... (Given)

$\therefore \tan\alpha = 1$

$\therefore \alpha = \frac{\pi}{4}$ **... (a)**

15. $\sim[p \vee (q \to \sim r)]$

$= (\sim p) \wedge \sim(q \to \sim r)$

$= \sim p \wedge (q \wedge \sim\sim r)$

$= \sim p \wedge (q \wedge r)$ **... (b)**

18. $$y = \log\left[\left(\frac{1 + \sin x}{1 - \sin x}\right)^{1/2}\right]$$

$$\therefore y = \frac{1}{2}[\log(1 + \sin x) - \log(1 - \sin x)]$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{1 + \sin x}(\cos x) - \frac{1}{1 - \sin x}(-\cos x)\right]$$

$$= \frac{1}{2}(\cos x)\left[\frac{1}{1 + \sin x} + \frac{1}{1 - \sin x}\right]$$

$$= \frac{\cos x}{\cos^2 x}$$

$$= \frac{1}{\cos x}$$

$$\therefore \left.\frac{dy}{dx}\right|_{x=\pi/3} = \frac{1}{\cos\frac{\pi}{3}} = \frac{1}{\left(\frac{1}{2}\right)} = 2$$ **... (c)**

20. $$y = \sin\left\{2 \cdot \tan^{-1}\sqrt{\frac{1 + x}{1 - x}}\right\}$$

Putting $\boldsymbol{x = \cos 2\theta}$,

$$\sqrt{\frac{1 + x}{1 - x}} = \ldots = \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

$$\therefore y = \left\{\sin 2 \cdot \tan^{-1}\left[\tan\left(\frac{\pi}{2} - \theta\right)\right]\right\}$$

$$= \sin\left\{2\left(\frac{\pi}{2} - \theta\right)\right\}$$

$$= \sin(\pi - 2\theta)$$

$$= \sin 2\theta$$

$$= \sqrt{1 - \cos^2 2\theta}$$

$$= \sqrt{1 - x^2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{1 - x^2}} \times \frac{d}{dx}(1 - x^2)$$

$$= \frac{1}{2\sqrt{1 - x^2}} \times (-2x)$$

$$= \frac{-x}{\sqrt{1 - x^2}}$$ **... (a)**

26. D.E. : $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ $\therefore \frac{y\,dy}{\sqrt{1-y^2}} = dx$

$\therefore \int \frac{-2y}{\sqrt{1-y^2}}\,dy = -2\int dx$

$\therefore 2\sqrt{1-y^2} = -2x + 2c$

$\therefore \sqrt{1-y^2} = -x + c$

$\therefore$ Sq. : $1 - y^2 = x^2 - 2cx + c^2$

$\therefore x^2 + y^2 - 2cx + (c^2 - 1) = 0$

$\therefore 2g = -2c, 2f = 0, k = c^2 - 1$

$\therefore C \equiv (-g, -f) \equiv (c, 0)$,

$r = \sqrt{g^2 + f^2 - k} = \sqrt{c^2 + 0^2 - (c^2 - 1)} = 1$

$\therefore r = 1$, fixed; $C \equiv (c, 0)$, moves on X-axis as c changes.

... (a)

27. $I = \int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx = \int_0^1 \sqrt{\frac{1-x}{1+x}\cdot\frac{1-x}{1-x}}\,dx$

$= \int_0^1 \frac{1-x}{\sqrt{1-x^2}}\,dx$

$= \int_0^1 \left[\frac{1}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}\right]dx$

$= [\sin^{-1}x + \sqrt{1-x^2}]_0^1$

$= (\sin^{-1}1 + 0) - (\sin^{-1}0 + 1)$

$= \frac{\pi}{2} - (0 + 1)$

$= \frac{\pi}{2} - 1$ **... (b)**

32. At the point $P(3, 4)$, $x = 3$

$\therefore$ slope of normal to the curve $y = f(x)$ at P is

$m_N = \frac{-1}{f'(3)} = \tan\frac{3\pi}{4} = -\cot\frac{\pi}{4} = -1$

$\therefore \frac{-1}{f'(3)} = -1$

$\therefore f'(3) = 1$ **... (d)**

33. $P(A \cup B) = 0.7$, $P(A \cap B) = 0.2$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A) + P(B) = P(A \cup B) + P(A \cap B)$

$= 0.7 + 0.2$

$= 0.9$

$\therefore P(A') + P(B') = [1 - P(A)] + [1 - P(B)]$

$= 2 - [P(A) + P(B)]$

$= 2 - 0.9$

$= 1.1$ **... (a)**

34. $\because$ the given limit exists as $x - 1$

$\therefore$ numerator has a factor $(x - 1)$

$\therefore$ its value at $x = 1$ is 0

$\therefore$ at $x = 1$, $x^2 - ax + b = 0$

$\therefore (1)^2 - a(1) + b = 0$

$\therefore 1 - a + b = 0$ **... (1)**

$\because \lim_{x \to 1} \frac{x^2 - ax + b}{x - 1} = 5$

$\therefore$ Using L' Hôpital's Rule on L.H.S.,

$\lim_{x \to 1} \frac{2x - a}{1} = 5$

$\therefore 2(1) - a = 5$

$\therefore a = -3$

$\therefore$ from (1), $1 - (-3) + b = 0$ $\therefore b = -4$

$\therefore a + b = -3 - 4 = -7$ **... (b)**

35. $\because y = \cos(\sin x^2)$ **... (1)**

$\therefore$ at $x = \sqrt{\frac{\pi}{2}}$,

$y = \cos\left(\sin\frac{\pi}{2}\right) = \cos 1$

(1) $\Rightarrow \frac{dy}{dx} = 2x \cdot \cos x^2 \cdot [-\sin(\sin x^2)]$

$= -2x \cdot \cos x^2 \cdot \sin(\sin x^2)$

$\therefore \left.\frac{dy}{dx}\right|_{x=\sqrt{\pi/2}} = -2\cdot\sqrt{\frac{\pi}{2}}\cdot\cos\frac{\pi}{2}\cdot\sin\left(\sin\frac{\pi}{2}\right)$

$= -2\cdot\sqrt{\frac{\pi}{2}}\cdot(0)\cdot\sin 1$

$= 0$ **... (a)**

37. Cartesian $\equiv (-2\sqrt{3}, 2) \equiv (r\cdot\cos\theta, r\cdot\sin\theta)$

$\therefore r = \sqrt{x^2 + y^2} = \sqrt{12 + 4} = 4$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \tan^{-1}\left(-\tan\frac{\pi}{6}\right)$

$= \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left(\tan\frac{5\pi}{6}\right)$

$= \frac{5\pi}{6}$

$\therefore$ Polar $\equiv \left(4, \frac{5\pi}{6}\right)$ **... (b)**

40. $\because y^{1/m} + y^{-1/m} = 2x$, ... $x \neq 1$ **... (1)**

$\therefore$ Squaring both sides,

$y^{2/m} + y^{-2/m} + 2 = 4x^2$

$\therefore$ adding **(–4)** to both sides

$y^{2/m} + y^{-2/m} - 2 = 4x^2 - 4$

$\therefore\ (y^{1/m} - y^{-1/m})^2 = 4\,(x^2 - 1)$

$\therefore\ y^{1/m} - y^{-1/m} = 2\sqrt{x^2 - 1}$ ... **(2)**

$\therefore$ **(1)** + **(2)** $\Rightarrow 2y^{1/m} = 2x + 2\sqrt{x^2 - 1}$

$\therefore\ y^{1/m} = x + \sqrt{x^2 - 1}$

$\therefore\ y = (x + \sqrt{x^2 - 1})^m$

$\therefore\ \log y = m \cdot \log (x + \sqrt{x^2 - 1})$

$\therefore$ **Differentiating** b.s. w.r.t. x,

$$\frac{1}{y}\cdot\frac{dy}{dx} = m\cdot\frac{1}{x + \sqrt{x^2 - 1}}\cdot\left[1 + \frac{1}{2\sqrt{x^2 - 1}}\cdot 2x\right]$$

$$\therefore\ \frac{dy}{dx} = \frac{my}{x + \sqrt{x^2 - 1}} \times \frac{\sqrt{x^2 - 1} + x}{\sqrt{x^2 - 1}}$$

$$\therefore\ \sqrt{x^2 - 1}\cdot\frac{dy}{dx} = my$$

$\therefore$ Squaring b.s. : $(x^2 - 1)\cdot\left(\frac{dy}{dx}\right)^2 = m^2y^2$ ... **(c)**

42. $(3, 2), 45^{\circ},\ L_1 : 1x - 2y - 3 = 0$

$\therefore\ m_1 = -(1)/(2) = 1/2$

Required lines $L :\ y - 2 = m\,(x - 3)$ ... **(1)**

$\because\ m \angle (L, L_1) = 45^{\circ}$

$$\therefore\ \tan 45^{\circ} = \left|\frac{m - m_1}{1 + mm_1}\right| = 1$$

$$\therefore\ \left|\frac{m - \frac{1}{2}}{1 + m\cdot\frac{1}{2}}\right| = 1 \qquad \therefore\ |\,2m - 1\,| = |\,m + 2\,|$$

$\therefore$ squaring : $4m^2 - 4m + 1 = m^2 + 4m + 4$

$\therefore\ 3m^2 - 8m - 3 = 0$

$\therefore\ (m - 3)\,(3m + 1) = 0$

$\therefore\ m = \mathbf{3},\ -\frac{\mathbf{1}}{\mathbf{3}}$

$\therefore$ from **(1)**, the required lines are

$y - 2 = 3\,(x - 3)$ and $y - 2 = -\frac{1}{3}\,(x - 3)$

$\therefore\ y - 2 = 3x - 9$ and $3y - 6 = -x + 3$

$\therefore\ \mathbf{3x - y - 7} = 0$ and $\mathbf{x + 3y - 9} = 0$ ... **(b)**

44. Required line $L \to P\,(1, 2, 3)$

$L \parallel E_1 : 1x - 1y + 2z = 5$

$L \parallel E_2 : 3x + 1y + 1z = 6$

$\overline{n_1} \equiv (1,\ -1,\ 2),\ \overline{n_2} \equiv (3,\ 1,\ 1)$

$$\therefore\ \overline{n_1} \times \overline{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix} = -3\hat{i} - 5\hat{j} + 4\hat{k}$$

$\equiv (-3, -5, 4)$

$\therefore$ d.R.s. of line L are $(-3, -5, 4)$ and it passes through $(1, 2, 3)$

$\therefore$ its cartesian equations are

$$\frac{x - 1}{-3} = \frac{y - 2}{-5} = \frac{z - 3}{4}$$ ... **(d)**

45. Solving $3x - 4y = 7$ and $2x - 3y = 5$,

centre $\equiv (1, -1)$

$\because$ area $\pi r^2 = 49\pi \quad \therefore\ r = 7$

$\therefore\ \odot : (x - 1)^2 + (y + 1)^2 = 7^2$

$\therefore\ x^2 + y^2 - 2x + 2y - 47 = 0$ ... **(a)**

47. $$I = \int \frac{\sin\left(\frac{5x}{2}\right)}{\sin\left(\frac{x}{2}\right)}\,dx$$

Putting $\frac{x}{2} = \theta$, i.e., $dx = 2d\theta$

$$I = \int \frac{\mathbf{\sin 5\theta}}{\sin\theta}\,d\theta$$

$$= \int \frac{\sin\theta\cdot\mathbf{(2\cos 2\theta + 2\cos 4\theta + 1)}}{\sin\theta}\cdot 2d\theta$$

$$= 2\cdot\int (1 + 2\cos 2\theta + 2\cos 4\theta)\,d\theta$$

$$= 2\left[\theta + 2\left(\frac{\sin 2\theta}{2}\right) + 2\left(\frac{\sin 4\theta}{4}\right)\right] + c,\ \ldots\boldsymbol{x} = \mathbf{2\theta}$$

$= 2\theta + 2\sin 2\theta + \sin 4\theta + c$

$= x + 2\sin x + \sin 2x + c$ ... **(Ans.)**

Note : None of the given option matches the answer.

48. Points $(2, 3, 1)$ and $(4, -5, 3)$ **both** satisfy **only** option (d).

$\therefore$ correct option is **(d)**.

49. $I = \int e^{x^2}\cdot x^3 dx$

$$= \frac{1}{2}\int x^2\cdot e^{x^2}\cdot(2x)\,dx$$

$$= \frac{1}{2}\int (t\cdot e^t)\,dt, \quad \ldots t = x^2$$

$$= \frac{1}{2}\,[t\,(e^t) - \int (e^t)\,(1)\,dt]$$

$$= \frac{1}{2}\,(t - 1)\cdot e^t$$

$$= \frac{1}{2}(x^2 - 1)\cdot e^{x^2}$$

$= e^{x^2}\cdot[f(x) - c]$ ... **(Given)**

$\therefore f(x) = \frac{1}{2}(x^2 - 1) + c$

$\because f(1) = 0 \quad \therefore \frac{1}{2}(0) + c = 0 \quad \therefore c = 0$

$\therefore f(x) = \frac{1}{2}(x^2 - 1) + 0$

$\therefore f(2) = \frac{1}{2}(4 - 1) = \frac{3}{2}$ **... (c)**

50. p : Payment is made,

q : Work is finished in time

$\therefore \sim(p \leftrightarrow q) = (p \wedge \sim q) \vee (\sim p \wedge q)$ **... (d)**