OLYMPIAD

NUMBER SYSTEM

CLASS 9

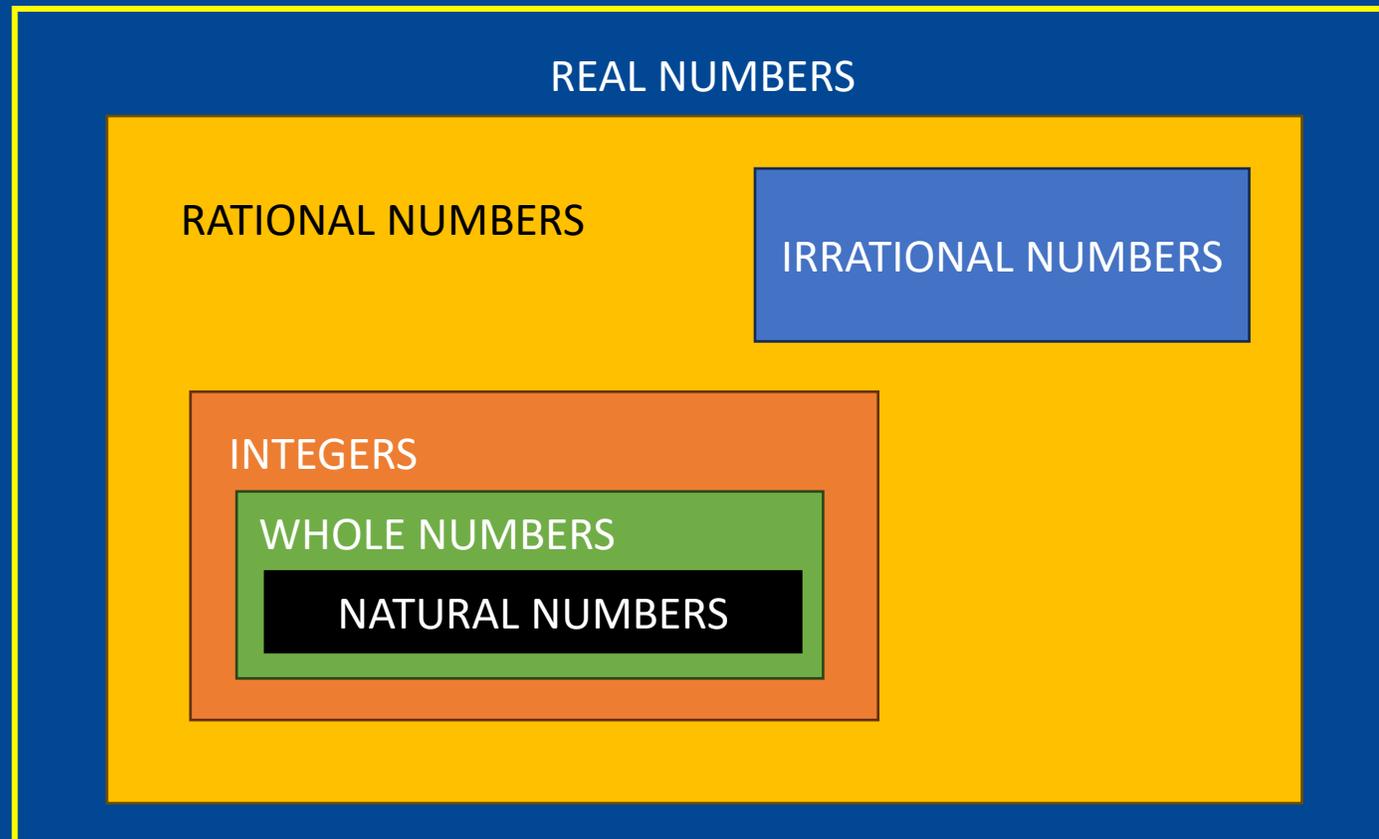OLYMPIAD CLASS 9

INVINCIBLE

CLASS MATHS

OLYMPIAD CLASS 9

Numbers

Numbers are <mark>mathematical objects used to represent quantities.</mark>

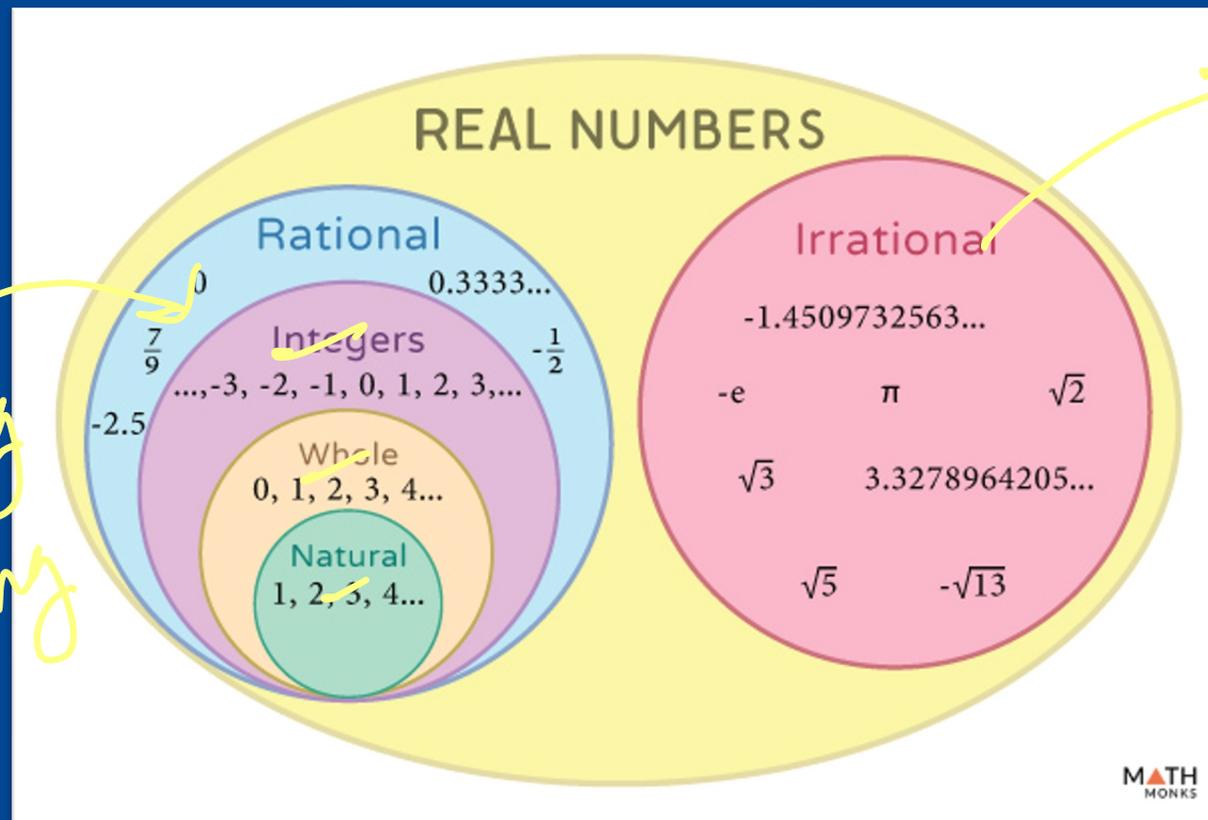**They can be classified into different types based on their properties. Here are some common types of numbers:**

REAL NUMBERS

RATIONAL NUMBERS

IRRATIONAL NUMBERS

INTEGERS

WHOLE NUMBERS

NATURAL NUMBERS

Real Numbers (R)

Real numbers can be defined as the union of both rational and irrational numbers.

Symbol : R

REAL NUMBERS

Rational
$\frac{7}{9}$ 0   0.3333...
-2.5   Integers $-\frac{1}{2}$
...,-3, -2, -1, 0, 1, 2, 3,...
Whole
0, 1, 2, 3, 4...
Natural
1, 2, 3, 4...

Irrational
-1.4509732563...
-e   π $\sqrt{2}$
$\sqrt{3}$ 3.3278964205...
$\sqrt{5}$ $-\sqrt{13}$

non-terminating

terminating
re-curring

Natural Numbers (N) :

Natural numbers are known as counting numbers that contain the positive integers from **1 to infinity**.

Symbol : N

Example : N = {1, 2, 3, 4, 5, ...}

Whole Numbers(W) :

Whole numbers are known as non-negative integers and it does not include any fractional or decimal part.

Symbol : W

Example : W = {0,1, 2, 3, 4, 5, ...}

Integers (Z) :

Integers are the set of all whole numbers but it includes a negative set of natural numbers also.

Symbol : Z

Example : Z = { -3, -2, -1, 0, 1, 2, 3}

Rational Numbers (Q) :

- The numbers of the form m/n are called rational numbers.
- Here, m and n are integers but n is not zero.

Symbol : Q

Example : 1/2, -3/4, 7, etc.

💡 TIP – Recurring or Terminating

$$\frac{4}{1} \neq \frac{p}{q}$$

$$\frac{p}{q} \qquad q \neq 0$$

$$q \Rightarrow 0$$

Irrational Numbers (P) :

- Irrational numbers are real numbers that cannot be represented as simple fractions.

- An irrational number cannot be expressed as a ratio, such as p/q, where p and q are integers,

Symbol : P

Example : √2, √5, π , -e , -1.4556871….. etc.

💡 TIP – Non-Recurring or Non-Terminating

Apart from the above, there exist other numbers :

Even Numbers :

The numbers which are exactly divisible by 2, are called even numbers. These can be positive or negative integers such as -42, -36, -12, 2, 4, 8 and so on.

Example :

Even Numbers Chart 1 - 100

2	12	22	32	42	52	62	72	82	92
4	14	24	34	44	54	64	74	84	94
6	16	26	36	46	56	66	76	86	96
8	18	28	38	48	58	68	78	88	98
10	20	30	40	50	60	70	80	90	100

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Odd Numbers::

The numbers which are not exactly divisible by 2, are called odd numbers. These can be both positive and negative integers such as -3, -15, 7, 9, 17, 25 and so on.

Example :

Odd Numbers Chart 1 - 100

1	11	21	31	41	51	61	71	81	91
3	13	23	33	43	53	63	73	83	93
5	15	25	35	45	55	65	75	85	95
7	17	27	37	47	57	67	77	87	97
9	19	29	39	49	59	69	79	89	99

Prime Numbers:

Prime numbers are the numbers that have two factors only. (i.e.,) 1 and the number itself.

In other words, the number which is divided by 1 and the number itself is called prime numbers.

For example: 2, 3, 5, 7, 11, etc.

NOTE:
1 ⟹ neither prime nor composite

2	3	5	7	11	13	17	19	23	29
31	37	41	43	47	53	59	61	67	71
73	79	83	89	97	101	103	107	109	113
127	131	137	139	149	151	157	163	167	173
179	181	191	193	197	199	211	223	227	229
233	239	241	251	257	263	269	271	277	281
283	293	307	311	313	317	331	337	347	349
353	359	367	373	379	383	389	397	401	409
419	421	431	433	439	443	449	457	461	463
467	479	487	491	499	503	509	521	523	541

Composite Numbers :

1, 2 → 2 ⟹ even prime

A composite number is a number that has more than two factors.

For example, 4 is a composite number, as the number 4 is divisible by 1, 2, and 4.

Other examples of composite numbers are 6, 8, 9, 10, and so on.
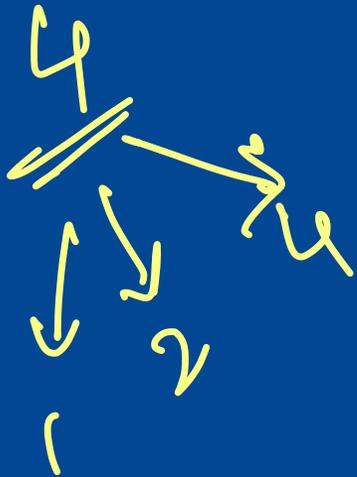
Composite numbers from 1 to 100

4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, 32,
33, 34, 35, 36, 38, 39, 40, 42, 44, 45, 46, 48, 49, 50, 51, 52, 54, 55,
56, 57, 58, 60, 62, 63, 64, 65, 66, 68, 69, 70, 72, 74, 75, 76, 77, 78,
80, 81, 82, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100

Perfect Square Numbers

- A perfect square is a number that is the square of an integer.

- In other words, a number is a perfect square if you can express it as n^2, where n is an integer.

$$4 = 2^2 \qquad 16 = 4^2 \qquad 81 = (\pm 9)^2$$

Surds

radical sign

power of surd. $\sqrt[n]{a}$ ← radicand

$n \neq 1$

surd $\boxed{\sqrt{5}}$ → irrational

NOTE:

$\sqrt[n]{a} = $ integer:

$\sqrt{16} = 4$

Rationalization of Denominators

$$\frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{5\sqrt{2}}{2}$$

NOTE: $\sqrt{n} \times \sqrt{n} = (\sqrt{n})^2 = n$

$$(a+b)(a-b) = a^2 - b^2$$

Rationalization of Denominators

$$\frac{5}{(\sqrt{3}-\sqrt{2})} \times \frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3}+\sqrt{2})} = \frac{5(\sqrt{3}+\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{5(\sqrt{3}+\sqrt{2})}{3-2}$$

$$= 5(\sqrt{3}+\sqrt{2})$$

Multiply and divide by conjugate value

$$\sqrt{3}-\sqrt{2}$$

$$\text{Conjugate} = \sqrt{3}+\sqrt{2}$$

If $2^{x-3} \cdot 3^{2x-8} = 36$, then the value of x is _____.

(A) 2   (B) 5   (C) 3   (D) 1

$$36 = 2^2 \cdot 3^2$$

$$2^2 \cdot 3^2 = 2^{x-3} \cdot 3^{2x-8}$$

$$x - 3 = 2 \implies x = 5$$

$$2x - 8 = 2 \implies 2x = 10$$
$$x = 5$$

2	36
2	18
3	9
3	3
	1

If $N = \dfrac{\sqrt{\sqrt{5}+2} + \sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}} - \sqrt{3-2\sqrt{2}}$, then N equals _____.

(A) 1

(B) $2\sqrt{2}-1$

(C) $\dfrac{\sqrt{5}}{2}$

(D) $\dfrac{2}{\sqrt{\sqrt{5}+1}}$

$\sqrt{2} - (\sqrt{2}-1) = \sqrt{2} - \sqrt{2}+1 = 1$

$$A = \sqrt{\sqrt{5}+2} + \sqrt{\sqrt{5}-2}$$

$$A^2 = \left(\sqrt{\sqrt{5}+2} + \sqrt{\sqrt{5}-2}\right)^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$A = \sqrt{2}\sqrt{\sqrt{5}+1}$$

$$A^2 = \sqrt{5}+2 + \sqrt{5}-2 + 2\sqrt{(\sqrt{5}+2)(\sqrt{5}-2)}$$

$$B = \sqrt{3-2\sqrt{2}}$$

$$= \sqrt{(\sqrt{2})^2 + (1)^2 - 2\sqrt{2}}$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$2 \times \sqrt{2} \times 1$$

$$= \sqrt{(\sqrt{2}-1)^2} = (\sqrt{2}-1)$$

$$(a+b)(a-b) = a^2 - b^2$$

$$A^2 = 2\sqrt{5} + 2\sqrt{5-4}$$

$$A^2 = 2\sqrt{5} + 2 = 2(\sqrt{5}+1)$$

Express the mixed recurring decimal $1.2\overline{7}$ in the form $\frac{p}{q}$.

(A) $\frac{8}{11}$

(B) $\frac{14}{11}$

(C) $\frac{14}{25}$

(D) $\frac{8}{17}$

100

$1.2727\ldots\ldots = x$

$127.2727\ldots = 100x$

$100x - x = 127.2727\ldots$
$\qquad\qquad\qquad - 1.2727$

$99x = 126$

$x = \frac{126}{99} = \frac{42}{33}$

$\qquad\quad = \frac{14}{11}$

1000

$3.\overline{271} = x$

$3271.271\ldots = 1000x$

If $x = (5 - 2\sqrt{6})$, then $\left(x^2 + \dfrac{1}{x^2}\right) = $ _____.

(A) 58   (B) 0   (C) 98   (D) 12

$$x = 5 - 2\sqrt{6}$$

$$x^2 = (5 - 2\sqrt{6})^2$$

$$= 5^2 + (2\sqrt{6})^2 - 2 \times 5 \times 2\sqrt{6}$$

$$= 25 + 24 - 20\sqrt{6}$$

$$= 49 - 20\sqrt{6}$$

$$\frac{1}{x^2} = \frac{1}{49 - 20\sqrt{6}} \times \frac{49 + 20\sqrt{6}}{49 + 20\sqrt{6}}$$

$$= \frac{49 + 20\sqrt{6}}{49^2 - (20\sqrt{6})^2}$$

$$2401 - 2400$$

$$49 \times 49$$
$$2401$$

$$\Rightarrow \frac{1}{x^2} = 40 + 20\sqrt{6} \quad \boxed{98}$$

$$x^2 + \frac{1}{x^2} = 49 - 20\sqrt{6} + 40 + 20\sqrt{6}$$

$$\frac{7\sqrt{3}}{(\sqrt{10}+\sqrt{3})} - \frac{2\sqrt{5}}{(\sqrt{6}+\sqrt{5})} - \frac{3\sqrt{2}}{(\sqrt{15}+3\sqrt{2})} =$$

(A) 1   (B) 2   (C) $\frac{1}{2}$ (D) 3

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} \times \frac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}} = \frac{7\sqrt{3}(\sqrt{10}-\sqrt{3})}{10-3}$$

$$= \frac{7\sqrt{3}(\sqrt{10}-\sqrt{3})}{7} = \sqrt{30}-3$$

$$\frac{2\sqrt{5}}{(\sqrt{6})+\sqrt{5}} = \frac{2\sqrt{5}(\sqrt{6}-\sqrt{5})}{6-5}$$

$$= 2\sqrt{30}-10$$

$$\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} = \frac{3\sqrt{2}(\sqrt{15}-3\sqrt{2})}{15-18}$$

$$= \frac{3\sqrt{2}(\sqrt{15}-3\sqrt{2})}{-3} = -(\sqrt{30}-6) = 6-\sqrt{30}$$

$$\sqrt{30}-3 - 2\sqrt{30}+10-6+\sqrt{30}$$

$$-3+4 = \boxed{1}$$

The rationalising factor of $\sqrt[5]{a^2 b^3 c^4}$ is _____.

(A) $\sqrt[5]{a^3 b^2 c}$

(B) $\sqrt[4]{a^3 b^2 c}$

(C) $\sqrt[3]{a^3 b^2 c}$

(D) $\sqrt{a^3 b^2 c}$

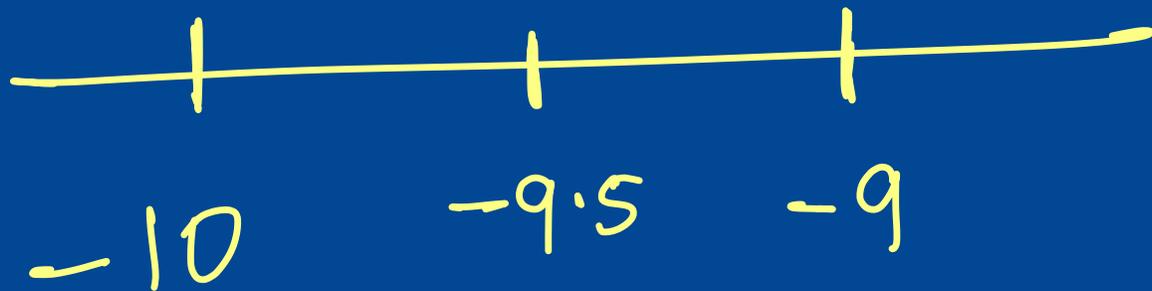$\sqrt{2} \times \sqrt{2} = 2$

$\sqrt[5]{a^5} = a$

$\sqrt[5]{a^2 b^3 c^4} \times \sqrt[5]{a^3 b^2 c^1} = abc$

Rational number $\dfrac{-19}{2}$ lies between consecutive integers _____.

(A) −2 and −1
(B) −7 and −8
(C) −6 and −7
(D) −9 and −10
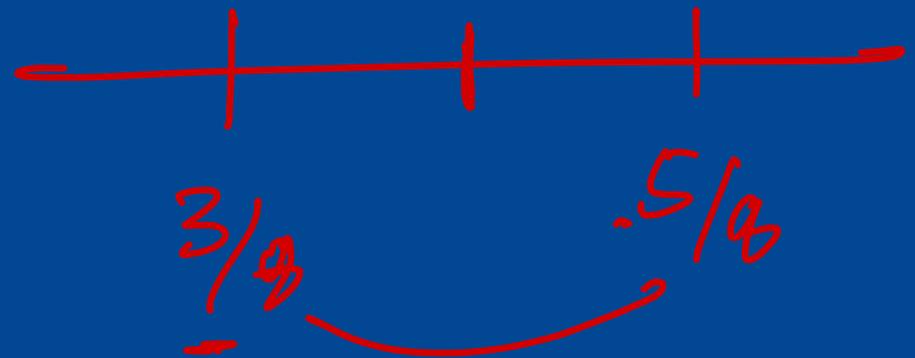
$$\dfrac{-19}{2} = \boxed{-9.5}$$

−10   −9.5   −9

An irrational number between $\frac{3}{8}$ and $\frac{5}{8}$ is _____.

(A) $\frac{1}{2}\left(\frac{3}{8} + \frac{5}{8}\right)$

(B) $\left(\frac{3}{8} \times \frac{5}{8}\right)$

(C) $\sqrt{\frac{3}{8} \times \frac{5}{8}}$

(D) None of these

The ascending order of the surds $\sqrt[3]{2}, \sqrt[6]{3}, \sqrt[9]{4}$ is _____

(A) $\sqrt[9]{4}, \sqrt[6]{3}, \sqrt[3]{2}$ (B) $\sqrt[9]{4}, \sqrt[3]{2}, \sqrt[6]{3}$

(C) $\sqrt[3]{2}, \sqrt[6]{3}, \sqrt[9]{4}$ (D) $\sqrt[6]{3}, \sqrt[9]{4}, \sqrt[3]{2}$

$3\sqrt{2} \qquad \sqrt[6]{3} \qquad \sqrt[9]{4}$

$\sqrt[3]{2} = 2^{1/3 \times 6/6} = 2^{6/18} = (2^6)^{1/18} = 64^{1/18}$

$3, 6, 9$

$3 = 1 \times 3$
$6 = 2 \times 3$
$9 = 3 \times 3$

$3 \times 2 \times 3$

$\sqrt{18}$

$\sqrt[6]{3} = 3^{1/6 \times 3/3} = (3^3)^{1/18} = 27^{1/18}$

$\sqrt[9]{4} = 4^{1/9 \times 2/2} = 4^{2/18} = 16^{1/18}$

$16 < 27 < 64$

The greater number among $\sqrt{17} - \sqrt{12}$ and $\sqrt{11} - \sqrt{6}$ is _____.

(A) $\sqrt{17} - \sqrt{12}$

(B) $\sqrt{11} - \sqrt{6}$

(C) Both are equal

(D) Can't be compared

$$\sqrt{17} - \sqrt{12} \, > \, \sqrt{11} - \sqrt{6}$$

$$\left(\sqrt{17} + \sqrt{6}\right)^2 > \left(\sqrt{11} + \sqrt{12}\right)^2$$

$$17 + 6 + 2\sqrt{102} > 11 + 12 + 2\sqrt{132}$$

$$23 + 2\sqrt{102} > 23 + 2\sqrt{132}$$

$$\sqrt{102} > \sqrt{132}$$

$$102 > 132$$

The value of x, if $5^{2x-1} = 25^{x-1} + 100$ is ___.

(A) 8   (B) 5   (C) 2   (D) 0

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{m \times n}$$

$$5^{2x} = 25 \times 25$$

$$(5^2)^x = 25^2 \Rightarrow 25^x = 25^2$$

$$25^{x-1} = \frac{25^x}{25} = \frac{(5^2)^x}{25}$$

$$\frac{5^{2x}}{5} = \frac{25^x}{25} + 100 \Rightarrow \frac{5^{2x}}{5} - \frac{25^x}{25} = 100$$

$$\frac{5^{2x}}{5} - \frac{5^{2x}}{25} = 100$$

$$\Rightarrow 5^{2x}\left(\frac{1}{5} - \frac{1}{25}\right) = 100$$

$$\Rightarrow 5^{2x}\left(\frac{5-1}{25}\right) = 100$$

$$5^{2x} = \frac{100 \times 25}{4}$$

If $x = 2 - \sqrt{3}$, then the value of $x^2 + \dfrac{1}{x^2}$ and $x^2 - \dfrac{1}{x^2}$ respectively, are _____.

(A) $14,\ 8\sqrt{3}$

(B) $-14,\ -8\sqrt{3}$

(C) $14,\ -8\sqrt{3}$

(D) $-14,\ 8\sqrt{3}$

Which of the following statements is INCORRECT ?

(A)   Every integer is a rational number.
(B)   Every natural number is an integer.
(C)   Every natural number is a real number.
(D)   Every real number is a rational number.

If $\dfrac{3 + 2\sqrt{5}}{3 - 2\sqrt{5}} = (p + q\sqrt{5})$, then find the value of $11(p + q)$.

(A) 31

(B) −41

(C) −31

(D) −40

$$\frac{3 + 2\sqrt{5}}{3 - 2\sqrt{5}} \times \frac{3 + 2\sqrt{5}}{3 + 2\sqrt{5}}$$

$$= \frac{(3 + 2\sqrt{5})^2}{(3)^2 - (2\sqrt{5})^2}$$

$$= \frac{9 + 20 + 2 \times 3 \times 2\sqrt{5}}{9 - 20}$$

$(a+b)^2 = a^2 + b^2 + 2ab$

$$\frac{29 + 12\sqrt{5}}{-11} = p + q\sqrt{5}$$

$$\frac{-29}{11} + \left(\frac{12}{-11}\right)\sqrt{5}$$

p q

$$11(p+q) = 11\left(\frac{-29 - 12}{11}\right) = \boxed{-41}$$

The number $x = 1.242424\ldots\ldots$ can be expressed in the form $x = \dfrac{p}{q}$, where p and q are positive integers having no common factors. Then $p + q$ equals _____.

(A) 72

(B) 74

(C) 41

(D) 53

$$1.24\underline{24}\ldots\ldots = x$$

$$124.2424\ldots = 100x$$

$$100x - x = 124.2424 - 1.24$$

$$99x = 123$$

$$x = \frac{123}{99} = \frac{41}{33} = \frac{p}{q}$$

$$p + q = 41 + 33$$

$$= \boxed{74}$$

$5 > 4$

If x and y are positive real numbers, then which of the following is CORRECT?

(A) $x > y \Rightarrow -x > -y$ $-5 < -4$

(B) $x > y \Rightarrow -x < -y$ $-5 < -4$

(C) $x > y \Rightarrow \dfrac{1}{x} > \dfrac{1}{y}$ $\dfrac{1}{5} > \dfrac{1}{4}$ 0.25

(D) $x > y \Rightarrow \dfrac{1}{x} < \dfrac{-1}{y}$ $0.2 < -0.25$

B

If $x = 1 - \sqrt{2}$, then find the value of $\left(x - \dfrac{1}{x}\right)^2$.

(A) 2   (B) 3   (C) 4   (D) 5

$$\frac{1}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$$

$$= \frac{1+\sqrt{2}}{1-2} = = -(1+\sqrt{2})$$

$$\left(x - \frac{1}{x}\right)^2 = \left(1 - \sqrt{2} + 1 + \sqrt{2}\right)^2 = (2)^2 = \boxed{4}$$

The value of

$$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} +$$

$$\frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{9}}$$

is _____.

(A) 0   (B) 1   (C) 2   (D) 4

$$\left(\frac{1}{\sqrt{1}+\sqrt{2}}\right) \times \frac{\sqrt{1}-\sqrt{2}}{\sqrt{1}-\sqrt{2}}$$

$$= \frac{\sqrt{1}-\sqrt{2}}{1-2} = \frac{\sqrt{1}-\sqrt{2}}{-1}$$

$$= \sqrt{2}-\sqrt{1}$$

$$\sqrt{2}-\sqrt{1} + \sqrt{3}-\sqrt{2} + \sqrt{4}-\sqrt{3} \dots$$

$$\dots + \sqrt{8}-\sqrt{7} + \sqrt{9}-\sqrt{8}$$

$$= -\sqrt{1} + \sqrt{9} = -1 + 3$$

$$= 2$$

Read the statements carefully.

Statement-1 : The product of a rational and an irrational number is an irrational number.

Statement-2 : Reciprocal of every rational number is a rational number.

Which of the following options hold?

(A) Both Statement-1 and Statement-2 are true.

(B) Statement-1 is true but Statement-2 is false.

(C) Statement-1 is false but Statement-2 is true.

(D) Both Statement-1 and Statement-2 are false.

Match the following.

	Column-I		Column-II
(a)	If $\dfrac{3}{x+8} = \dfrac{4}{6-x}$, then x is _____ .	(i)	3
(b)	If $\dfrac{2^{x-1} \cdot 4^{2x+1}}{8^{x-1}} = 64$, then x is _____ .	(ii)	$5^{5/2}$
(c)	If $4^x - 4^{x-1} = 24$, then $(2x)^x$ is _____ .	(iii)	-2
(d)	If $4^{x+1} = 256$, then x is _____ .	(iv)	1

(A) (a)→(i); (b)→(ii); (c)→(iii); (d)→(iv)
(B) (a)→(iii); (b)→(iv); (c)→(i); (d)→(ii)
(C) (a)→(i); (b)→(iv); (c)→(ii); (d)→(iii)
(D) (a)→(iii); (b)→(iv); (c)→(ii); (d)→(i)