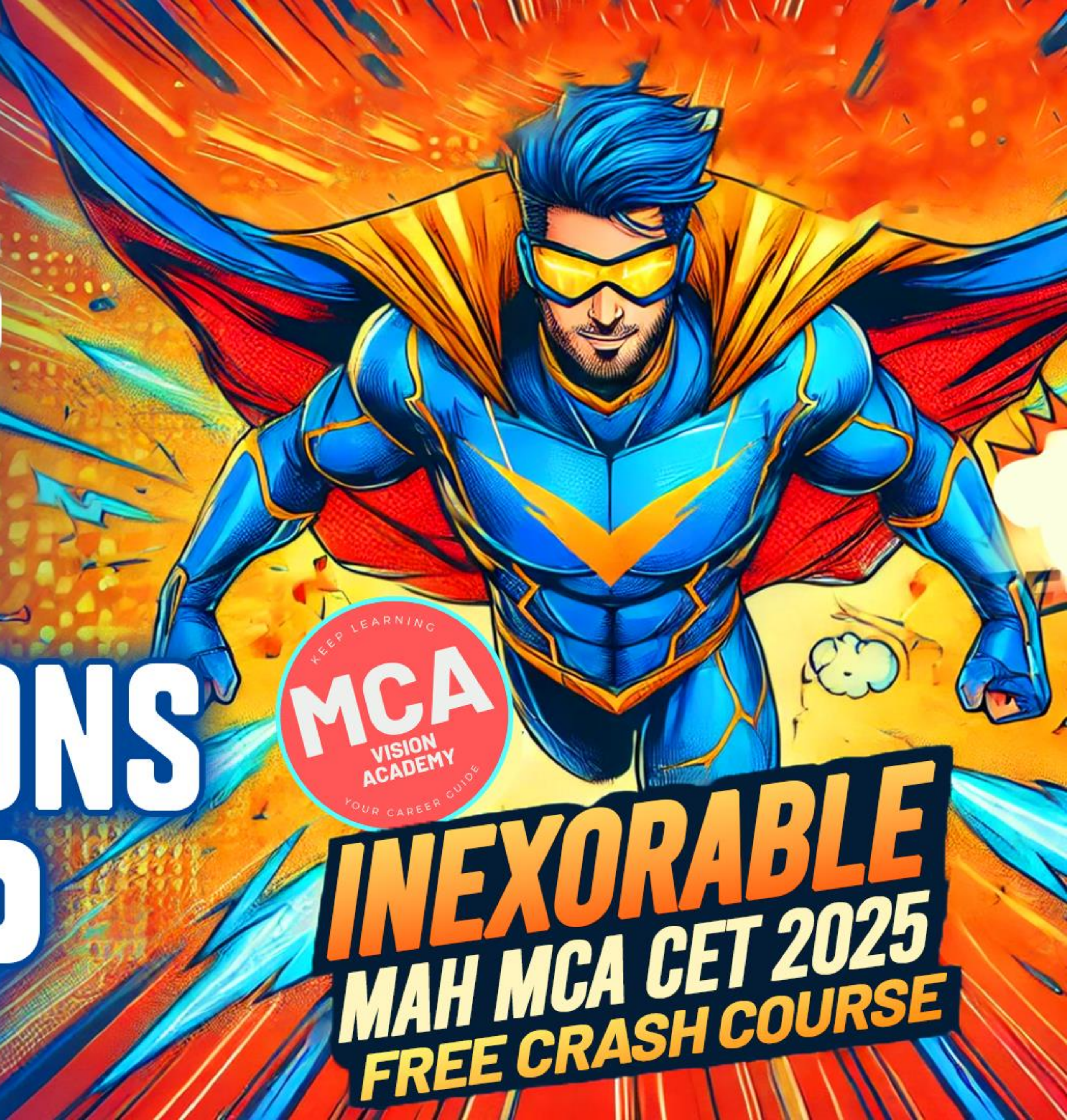
DAY 41

MCA CET 2025

MATHS

PROGRESSIONS

AP/GP/HP



Progressions

A sequence following a certain pattern is known as a progression.

Series.

2, 2, 2, 2,

$$3 \underbrace{\quad}_{+3} 6 \underbrace{\quad}_{+3} 9 \underbrace{\quad}_{+3} 12 \underbrace{\quad}_{+3} 15$$

Arithmetic progressions

$$a,\ a+d,\ a+2d,\ a+3d, \ldots\ldots$$

first term

d = common difference

$\therefore$ n^{th} term of an A.P.

$$\boxed{t_n = a + (n-1)d}$$

1, 2, 3, 4, 5 100

Sum of n terms in A.P.

$a,\ a+d,\ a+2d,\ \ldots\ldots\ a+(n-1)d$

$$S_n = \frac{n}{2}[t_1 + t_n]$$

$a + a + (n-1)d$

$2a + (n-1)d$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$2, 4, 8, 16, 32, \ldots \ldots \longrightarrow 2^1, 2^2, 2^3, 2^4, 2^5, \ldots$

Geometric Progression

r

$$\frac{\cancel{4}}{\cancel{2}} = \frac{\cancel{8}}{\cancel{4}} = \frac{\cancel{16}}{\cancel{8}} = \frac{2}{1}$$

r

$a_1, a_2, a_3, \ldots \ldots, a_n$

are in G.P. if.

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \ldots = \frac{a_n}{a_{n-1}} = r$$

where r = common ratio of G.P.

$a \longrightarrow 2, 4, 8, 16, \ldots$

$n = 18$

$a = 2$

$r = 2$

nth term of G.P.

$t_{18} = 2 \cdot 2^{17}$

$= 2^{18}$

$$t_n = ar^{n-1}$$

Sum of n terms of G.P.

$$S_n = \frac{a(r^n-1)}{r-1} \quad \text{where } r>1$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{where } r<1$$

Sum of infinite terms of G.P.

$$S_\infty = \frac{a}{1-r} \quad \text{where } |r|<1$$

Harmonic Progression

$2, 4, 6, 8, 10, \ldots$

$\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots$

$a_1, a_2, a_3, \ldots, a_n$ are in H.P. if

$\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots \frac{1}{a_n}$ are in A.P.

From start.

$$\boxed{n^{th} \text{ term of H.P. } (T_n) = \frac{a_1 a_2}{a_2 + (n-1)(a_1 - a_2)}}$$

n^{th} term from the $\boxed{\text{end}}$ of H.P.

$$T_n' = \frac{a_1 a_2 a_n}{a_1 a_2 - a_n (n-1)(a_1 - a_2)}$$

Important Note:

- $\frac{1}{T_n} + \frac{1}{T_n'} = \frac{1}{a_1} + \frac{1}{a_n}$
- Any term of HP cannot be zero.

Mean of a and b.

Arithmetic Mean (A) $= \frac{a+b}{2}$

Geometric Mean (G) $= \sqrt{ab}$

Harmonic Mean (H) $= \frac{2ab}{a+b}$

Relation between A, G and H

$$A > G > H$$

$$G^2 = AH$$

$$\therefore \quad G = \sqrt{AH}$$

Sum of n terms of Series.

1. Sum of n natural numbers

$$= 1 + 2 + 3 + 4 + \ldots\ldots + n$$

$$= \sum n = \frac{n(n+1)}{2}$$

2. Sum of squares of n natural nos.

$$1^2 + 2^2 + 3^2 + \cdots + n^2$$

$$= \sum n^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$

Sum of cubes of n natural nos.

$$= 1^3 + 2^3 + 3^3 + \dots + n^3$$

$$= \Sigma n^3$$

$$= \frac{n^2(n+1)^2}{4} = \left[\frac{n(n+1)}{2}\right]^2 = (\Sigma n)^2$$

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{n(n+1)}$$

$$= \frac{n}{n+1}$$

$$\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\frac{1}{3\cdot 4\cdot 5}+\cdots\frac{1}{n(n+1)(n+2)}$$

$$=\frac{1}{4}-\frac{1}{2(n+1)(n+2)}$$

$$1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \cdots + n(n+1)$$

$$= \frac{n(n+1)(n+2)}{3}$$

$r = \dfrac{-\frac{1}{2}}{\frac{1}{4}} = -\frac{1}{2} \times \frac{4}{1} = -2$

Which term of a series ¼ , - ½ , 1, … is $\boxed{-128}$?

(a) 9th

(b) 10th

(c) 11th

(d) 12th

$t_n = a \cdot r^{n-1}$

$-128 = \frac{1}{4} (-2)^{n-1}$

$-512 = (-2)^{n-1}$

$(-2)^9 = (-2)^{n-1}$

$9 = n - 1$

$n = 9 + 1$

$n = 10$

The value of $0.4\overline{23}$ is

(a) 419/990

(b) 419/999

(c) 417/990

(d) 417/999

$$\frac{4190}{9900}$$

$a = 1$

$r = \frac{1}{100}$

$$0.4 + 0.023 + 0.00023 + 0.0000023 + \cdots$$

$$= 0.4 + \left[\frac{23}{10^3} + \frac{23}{10^5} + \frac{23}{10^7}\right]$$

$$= 0.4 + \frac{23}{10^3}\left[1 + \frac{1}{10^2} + \frac{1}{10^4} + \cdots\right]$$

$$= 0.4 + \frac{23}{10^3}\left[\left(\frac{1}{10}\right)^0 + \left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^4 + \cdots\right]$$

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{100}} = \left(\frac{100}{99}\right)$$

$$= 0.4 + \frac{23}{1000} \times \frac{100}{99}$$

$$\frac{4}{10} + \frac{23}{990} = \frac{3960+230}{9900}$$

$180(n-2)$ $n \geq 3$

$180(2) = 360$

$180 \times 3 = 540$

$n(n-16) - 9(n-16) = 0$

$(n-9)(n-16) = 0$

9 16

The interior angles of a polygon are in AP. If the smallest angle be 120° and the common difference be 5, then the number of side is

$16 \times 9 = 144$

(a) 8

(b) 10

(c) 9

(d) 6

$S_n = \frac{n}{2}[2a + (n-1)d]$

$180(n-2) = \frac{n}{2}[240 + (n-1)5]$

$360(n-2) = n[240 + 5n - 5]$

$360n - 720 = 235n + 5n^2$

$5n^2 + 235n - 360n + 720 = 0$

$5n^2 - 125n + 720 = 0$

$n^2 - 25n + 144 = 0$

$n^2 - 16n - 9n + 144 = 0$

$a = p \quad d = q$

$S_{10} = 4S_5$

$S_n = \frac{n}{2}[2a + (n-1)d]$

If the sum of first 10 terms of an arithmetic progression with first term p and common difference q, is 4 times the sum of the first 5 terms, then what is the ratio of $\boxed{p : q}$?

(a) 1:2

(b) 1:4

(c) 2:1

(d) 4:1

$\frac{\cancel{10}}{\cancel{2}}[2p + 9q] = {}^{2}\cancel{4} \times \frac{\cancel{5}}{\cancel{2}}[2p + 4q]$

$2p + 9q = 4p + 8q$

$q = 2p$

$\frac{1}{2} = \frac{p}{q}$

$AP_1 \Rightarrow a=1 \quad d=1 \Rightarrow S_n = \frac{n}{2}[2+(n-1)1]$

$AP_2 \Rightarrow a=1 \quad d=2 \qquad = \frac{n}{2}[2+n-1]$

$AP_3 \Rightarrow a= \quad d=3 \qquad S_1 = \boxed{\frac{n}{2}[1+n]}$

The sum of n terms of three AP's is whose first term is 1 and common differences are 1, 2 and 3 are S_1, S_2 and S_3, respectively. Then, the true relation is

(a) $S_1 + S_3 = S_2$

~~(b)~~ $S_1 + S_3 = 2S_2$

(c) $S_1 + S_2 = 2S_3$

(d) $S_1 + S_2 = S_3$

$S_2 = \frac{n}{2}[2+(n-1)2]$

$= \frac{n}{\cancel{2}}[\cancel{2}+2n-\cancel{2}] = \frac{n}{\cancel{2}}[\cancel{2}n] = n^2$

$S_3 = \frac{n}{2}[2+3n-3] = \frac{n}{2}[3n-1]$

$S_1 + S_3 = \cancel{\frac{n}{2}} + \frac{n^2}{2} + \frac{3n^2}{2} - \cancel{\frac{n}{2}} = \frac{4n^2}{2} = \boxed{2n^2} = 2S_2$

$1, 2, 3, 4, 5, 6, \ldots$ 100

$2 \Rightarrow 2, 4, 6, 8, 10, \ldots 20 \ldots 100 \Rightarrow a = 2 \quad n = 50$

$t_n = 100$

S_n

The sum of integers from 1 to 100 that are divisible by 2 or 5 is

(a) 3000

(b) 3050

(c) 4050

(d) None of these

$S_n = \frac{50}{2}[2 + 100]$

$= 25 \times 102$

$= \boxed{2550}$

$2550 + 500$

$\boxed{5}, 10, \boxed{15}, 20, \boxed{25}, \ldots 100$

$5, 15, 25, \ldots 95$

$S_n = \frac{10}{2}[5 + 95]$

$= 5 \times 100 = 500$

$t_1 + t_3 = 12$

$t_1 \times t_2 = 24$

$\boxed{a-d}, a, a+d$

$6-2 = 4$

The sum of the first and third term of an arithmetic progression is 12 and the product of of first and second term is 24, then first term is

(a) 1 (b) 8 (c) 4 (d) 6

$a - \cancel{d} + a + \cancel{d} = 12 \Rightarrow 2a = 12$

$a = 6$

$(a-d)(a) = 24$

$(6-d)(6) = 24$

$6 - d = 4$

$d = 2$

$G = \sqrt{xy} \Rightarrow G^2 = xy$

If G be the geometric mean of x and y, then $\frac{1}{G^2 - x^2} + \frac{1}{G^2 - y^2}$ is equal to

(a) G^2

(b) $1/G^2$

(c) $2/G^2$

(d) $3G^2$

$$\frac{1}{xy - x^2} + \frac{1}{xy - y^2}$$

$$\frac{1}{x(y-x)} + \frac{1}{y(x-y)} = \frac{1}{x(y-x)} - \frac{1}{y(y-x)}$$

$$= \frac{y}{xy(y-x)} - \frac{x}{xy(y-x)} = \frac{\cancel{y-x}}{xy\cancel{(y-x)}}$$

$$= \frac{1}{xy} = \frac{1}{G^2}$$

$t_n = ar^{n-1}$

$16 = 1 \cdot r^4$

$16 = r^4$

$r = 2$

$2^0 \quad 2^1 \cdot 2^2 \quad 2^2 \quad 2^4$

If 1, x, y, z and $\boxed{16}$ are in geometric progression, then what is the value of x + y +z?

(a) 8 (b) 12 (c) 14 (d) 16

$2+4+8 = 14$