DAY 57

MCA CET 2025

MATHS

TRIGINOMETRIC FUNCTIONS & IDENTITIES

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Trigonometric Ratio

sin $\frac{O}{H}$ cosec   cos $\frac{A}{H}$ sec   tan $\frac{O}{A}$ cot

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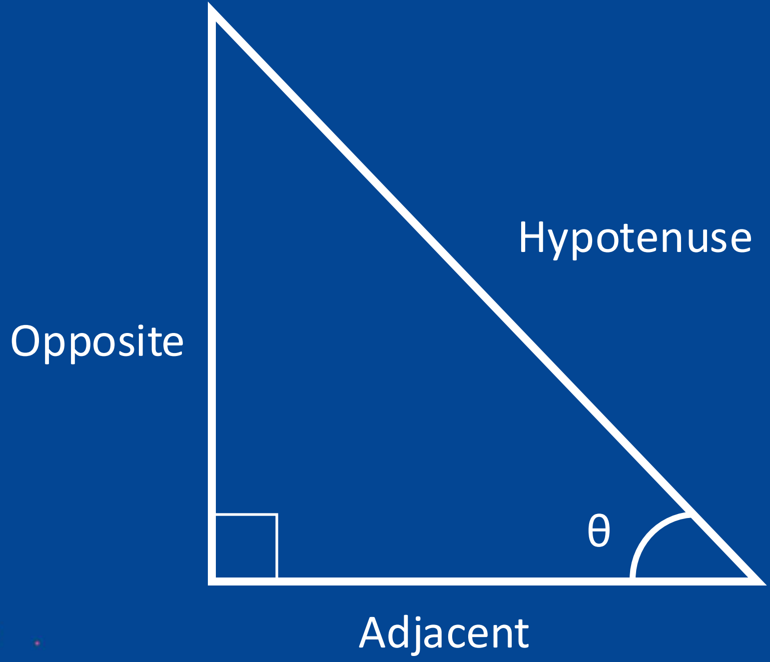
In right angled △ ABC.



$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$

$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$

$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{Opposite}}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{Hypotenuse}}{\text{Adjacent}}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{\text{Adjacent}}{\text{Opposite}}$

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Fundamental Relation Among Trigonometric Ratios

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$ and $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$ and $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ and $\cot\theta = \frac{1}{\tan\theta}$

From these you can also see that $\sin\theta \times \operatorname{cosec}\theta = 1$

$\cos\theta \times \sec\theta = 1$

and $\tan\theta \times \cot\theta = 1$

Two other significant relationships between the trigonometric ratios are:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Trigonometric Identities

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1. $\sin^2\theta + \cos^2\theta = 1$ $\rightarrow$ $\cos^2\theta = 1 - \sin^2\theta$

   $\rightarrow$ $\sin^2\theta = 1 - \cos^2\theta$

2. $1 + \tan^2\theta = \sec^2\theta$ $\rightarrow$ $\sec^2\theta - \tan^2\theta = 1$

   $\rightarrow$ $\sec^2\theta - 1 = \tan^2\theta$

3. $1 + \cot^2\theta = \text{cosec}^2\theta$ $\rightarrow$ $\text{cosec}^2\theta - \cot^2\theta = 1$

   $\text{cosec}^2\theta - 1 = \cot^2\theta$

If 5 tanθ = 4, then $\left[\dfrac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}\right]$ is equal to

(a) 0

(b) 1

(c) $\frac{1}{6}$

(d) 6

$5\tan\theta = 4$

$\tan\theta = \frac{4}{5}$

$\frac{\sin\theta}{\cos\theta} = \frac{4}{5}$

$$= \frac{\dfrac{5\boxed{\sin\theta}}{\boxed{\cos\theta}} - \dfrac{3\cancel{\cos\theta}}{\cancel{\cos\theta}}}{\dfrac{5\boxed{\sin\theta}}{\boxed{\cos\theta}} + \dfrac{2\cancel{\cos\theta}}{\cancel{\cos\theta}}} = \frac{\cancel{5}\times\frac{4}{\cancel{5}} - 3}{\cancel{5}\times\frac{4}{\cancel{5}} + 2} = \boxed{\frac{1}{6}}$$

GOLDEN RULE: Convert everything to $\sin\theta / \cos\theta$

The value of (1 + cotθ - cosecθ) (1 + tanθ + secθ) is

(a) 3

(b) 2

(c) -2

(d) -3

$$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$$

$$= \left(1 + \frac{\cos\theta - 1}{\sin\theta}\right)\left(1 + \frac{\sin\theta + 1}{\cos\theta}\right)$$

$$= \left(\frac{(\sin\theta + \cos\theta) - (1)}{\sin\theta}\right)\left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)$$

$$= \frac{\cancel{\sin^2\theta + \cos^2\theta} + 2\sin\theta\cdot\cos\theta - \cancel{1}}{\sin\theta\cdot\cos\theta} = \frac{2\cancel{\sin\theta\cdot\cos\theta}}{\cancel{\sin\theta\cdot\cos\theta}}$$

$$= 2$$

Trigonometric Ratios of Combined Angles

Sum and Difference of two angles

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sin (A + B): sin A cos B + cos A sin B

sin (A – B): sin A cos B – cos A sin B

cos (A + B): cos A cos B – sin A cos B

cos (A – B): cos A cos B + sin A cos B

tan (A + B): [tan A + tanB] / [1 – tan A tan B] $= \frac{t_1 + t_2}{1 - t_1 t_2}$

tan (A – B): [tan A – tan B] / [1 + tan A tan B] $= \frac{t_1 - t_2}{1 + t_1 t_2}$

$$\cot(A+B) = \frac{\cot A \cdot \cot B - 1}{\cot A + \cot B}$$

$$\cot(A-B) = \frac{\cot A \cdot \cot B + 1}{\cot A - \cot B}$$

* $\sin(A+B) \cdot \sin(A-B)$
$= \sin^2 A - \sin^2 B$
$= \cos^2 B - \cos^2 A$

* $\cos(A+B) \cdot \cos(A-B)$
$= \cos^2 A - \sin^2 B$
$= \cos^2 B - \sin^2 A$

Example

The value of $\sin\theta + \sin(\frac{2}{3}\pi + \theta) + \sin(\frac{4}{3}\pi + \theta)$ is

a) 0

b) 1

c) 2

d) 3

TRIGONOMETRIC ⇒ π = radians.

$\pi = 180°$

S | A

T | C

All silver tea cups

$\sin(\pi - \theta)$

$= \sin\theta + \sin\left(\pi - \frac{\pi}{3} + \theta\right) + \sin\left(\pi + \frac{\pi}{3} + \theta\right)$

$= \sin\theta + \sin\left(\pi - [\pi/3 - \theta]\right) + \sin\left(\pi + [\pi/3 + \theta]\right)$

$= \sin\theta + \sin(\underbrace{\pi/3 - \theta}_{A-B}) - \sin(\underbrace{\pi/3 + \theta}_{A+B})$

$= \sin\theta + \cancel{\sin\pi/3 \cdot \cos\theta} - \sin\theta \cdot \cos\pi/3 - \cancel{\sin\pi/3 \cdot \cos\theta}$

$- \sin\theta \cdot \cos\pi/3$

$= \sin\theta - \frac{\sin\theta}{2} - \frac{\sin\theta}{2} = \sin\theta - \left[\frac{\sin\theta}{2} + \frac{\sin\theta}{2}\right]$

$= \sin\theta - \sin\theta = 0$

Transformation of product into sum or difference

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$$2\sin A\cos B = \sin(A + B) + \sin(A - B)$$

$$2\cos A\sin B = \sin(A + B) - \sin(A - B)$$

$$2\cos A\cos B = \cos(A + B) + \cos(A - B)$$

$$2\sin A\sin B = \cos(A - B) - \cos(A + B)$$

Transformation of sum or difference into Product

ॐ

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\,.\,\cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\,.\,\sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\,.\,\cos\frac{C-D}{2}$$

$$\cos D - \mathrm{Cos}\, C = 2\sin\frac{C+D}{2}\,.\,\sin\frac{C-D}{2}$$

Transformation of 2θ in terms of θ

(I) $\sin(2\theta) = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta} = \dfrac{2t}{1+t^2}$

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(II) $\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

$= 2\cos^2\theta - 1 = \dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-t^2}{1+t^2}$

(III) $\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{2t}{1-t^2}$

(IV) $\cot(2\theta) = \dfrac{\cot^2\theta - 1}{2\cot\theta} = \dfrac{c^2-1}{2c}$

Transformation of θ in terms of θ/2

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I) $\sin\theta = 2\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta = \dfrac{2\tan\frac{1}{2}\theta}{1+\tan^2\frac{1}{2}\theta}$

II) $\cos\theta = \cos^2\frac{1}{2}\theta - \sin^2\frac{1}{2}\theta = 1 - 2\sin^2\frac{1}{2}\theta$

$= 2\cos^2\frac{1}{2}\theta - 1 = \dfrac{1-\tan^2\frac{1}{2}\theta}{1+\tan^2\frac{1}{2}\theta}$

III) $\tan\theta = \dfrac{2\tan\frac{1}{2}\theta}{1-\tan^2\frac{1}{2}\theta}$

IV) $\cot\theta = \dfrac{\cot^2\frac{1}{2}\theta - 1}{2\cot\frac{1}{2}\theta}$

Example

The value of $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}$ is

$2(1+\cos 8\theta)$

(a) – 2 cosθ

(b) cosθ

(c) 2cosθ

(d) None of the these

$\cos 2\theta = 2\cos^2\theta - 1$

$1+\cos 2\theta = 2\cos^2\theta$

$\sqrt{2+\sqrt{2+\sqrt{2\cdot 2\cdot \cos^2(4\theta)}}}$ $\rightarrow 2(4\theta)$

$= \sqrt{2+\sqrt{2+2\cos 4\theta}}$

$2(1+\cos 4\theta)$

$\sqrt{2+\sqrt{2(2\cdot\cos^2 2\theta)}}$

$= \sqrt{2+2\cdot\cos 2\theta}$

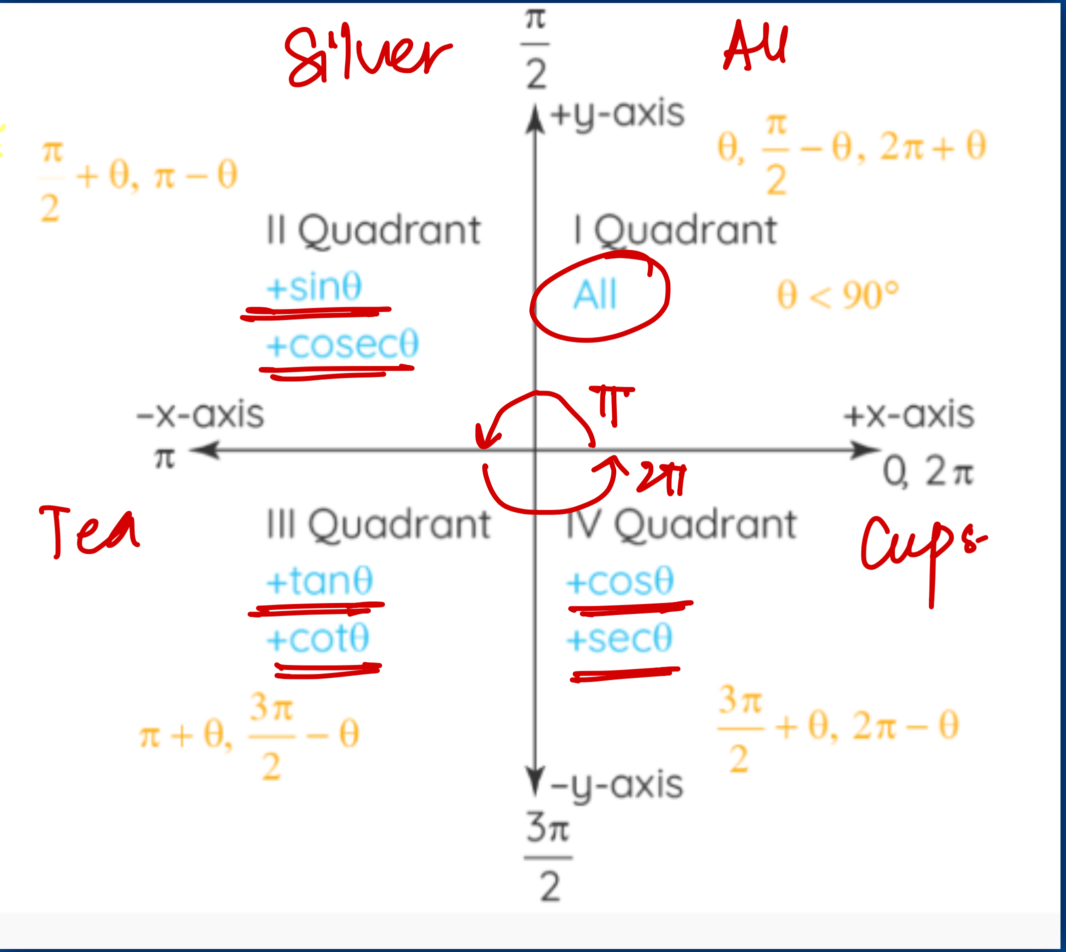
$= \sqrt{2(1+\cos 2\theta)}$

$= \sqrt{2\cdot 2\cdot \cos^2\theta}$

$= 2\cdot\cos\theta$

Trigonometric Ratios in Different Quadrants

II Quad



I Quad.

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Transformation of 3θ in terms of θ

i) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

ii) $\cos 3\theta = \underline{4\cos^3\theta} - \underline{3\cos\theta}$

iii) $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = \dfrac{3t - t^3}{1 - 3t^2}$

iv) $\cot 3\theta = \dfrac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta - 1} = \dfrac{3\cot\theta - \cot^3\theta}{1 - 3\cot^2\theta} = \dfrac{3c - c^3}{1 - 3c^2}$

$= \dfrac{c^3 - 3c}{3c^2 - 1}$

Example

The simplified form of cosA cos(60°- A) cos(60°+ A) is

$\cos^2 A - \sin^2 B$

(a) sin 3A

(b) cos 3A

(c) $\frac{1}{2}$ cos3A

(d) $\frac{1}{4}$ cos3A

$= \cos A \cdot [\cos^2 60 - \sin^2 A]$

$= \cos A \left[\frac{1}{4} - \sin^2 A\right] = \cos A \left[\frac{1}{4} - (1 - \cos^2 A)\right]$

$= \cos A \left[\frac{1}{4} - 1 + \cos^2 A\right] = \left[-\frac{3}{4} + \cos^2 A\right] \cdot \cos A$

$= \left[\frac{-3 + 4\cos^2 A}{4}\right] \cdot \cos A$

$\frac{4\cos^3 A - 3\cos A}{4} = \frac{\cos 3A}{4}$

Example

The value of tan5x tan3x tan2x is

(a) tan5x – tan3x

(b) tan5x – tan 3x – tan 2x

(c) tan5x + tan3x

(d) None of these

$\tan(3x + 2x)$

$\tan 5x = \tan(3x + 2x)$

$$\tan 5x = \frac{\tan 3x + \tan 2x}{1 - \tan 3x \cdot \tan 2x}$$

$\tan 5x(1 - \tan 3x \cdot \tan 2x) = \tan 3x + \tan 2x$

$\tan 5x - \tan 5x \cdot \tan 3x \cdot \tan 2x = \tan 3x + \tan 2x$

$\tan 5x - \tan 3x - \tan 2x =$

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$$\cot(A+B) = \frac{\cot A\cdot\cot B - 1}{\cot A + \cot B}.$$

cotx.cot2x – cot2x.cot3x – cot3x.cotx is equal to

(a) -1

(b) 0

~~(c)~~ 1

(d) $\frac{1}{2}$

$$= \cot x\cdot\cot 2x - \underbrace{\cot 3x}_{(2x+x)}\left(\cot 2x + \cot x\right)$$

$$= \cot x\cdot\cot 2x - \left[\frac{\cot 2x\cdot\cot x - 1}{\cancel{\cot 2x + \cot x}}\right]\left[\cancel{\cot 2x + \cot x}\right]$$

$$= \cancel{\cot x\cdot\cot 2x} - \cancel{\cot 2x\cdot\cot x} + 1$$

$$= 1$$

$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

The value of sin50° - sin70° + sin10° is

(a) 0

(b) 1

(c) $\frac{1}{2}$

(d) $\frac{1}{\sqrt{2}}$

$\sin(-\theta) = -\sin\theta$

$= 2\cos\left(\frac{50+70}{2}\right) \cdot \sin\left(\frac{50-70}{2}\right) + \sin 10$

$= 2\cos\left(\frac{\cancel{120}^{60}}{\cancel{2}}\right) \cdot \sin\left(\frac{-\cancel{20}^{10}}{\cancel{2}}\right) + \sin 10$

$= \cancel{2} \times \frac{1}{\cancel{2}} \times -\sin 10 + \sin 10$

$= \cancel{-\sin 10} + \cancel{\sin 10}$

$= 0$

$2\cos^2 x = 1 + \cos 2x$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\frac{1+\cos\left[2\left(x-\pi/3\right)\right]}{2}$

The value of $\cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(\cancel{\pi}^{\,x} - \frac{\cancel{2}\pi}{3}\right)$ is

$\frac{1+\cos\left[2\left(x+\pi/3\right)\right]}{2}$

(a) $\frac{3}{2}$

(b) $\frac{4}{3}$

(c) $\frac{3}{4}$

(d) $\frac{2}{3}$

$\cos(-)$

S A

T C

$= \frac{1}{2}\left[1 + \cos 2x + 1 + \cos\left[2\left(x+\pi/3\right)\right] + 1 + \cos\left[2\left(x-\pi/3\right)\right]\right]$

$= \frac{1}{2}\left[3 + \cos 2x + \cos\left[2x + \frac{2\pi}{3}\right] + \cos\left[2x - \frac{2\pi}{3}\right]\right]$

$= \frac{1}{2}\left[3 + \cos 2x + 2\cdot\cos 2x\cdot\cos\left(\pi - \frac{\pi}{3}\right)\right]$

$\Downarrow$

$-\cos \pi/3 = -\frac{1}{2}$

$= \frac{1}{2}\left[3 + \cancel{\cos 2x} - \cancel{2}\cdot\frac{1}{\cancel{2}}\cdot\cancel{\cos 2x}\right] = \frac{3}{2}$

If sin(x + 3α) = 3 sin (α - x) then

(a) tanx = tanα

(b) tanx = $\tan^2\alpha$

(c) tanx = $\tan^3\alpha$

(d) tanx = 3tanα

$\sin x \cdot \cos 3\alpha + \cos x \cdot \sin 3\alpha = 3[\sin\alpha \cdot \cos x - \cos\alpha \cdot \sin x]$

$= 3\sin\alpha \cdot \cos x - 3\cos\alpha \cdot \sin x$

$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$

$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$

$\sin x \cdot \cos 3\alpha + \cos x \cdot \sin 3\alpha - 3\sin\alpha \cdot \cos x + 3\cos\alpha \cdot \sin x = 0$

$\sin x(\cos 3\alpha + 3\cos\alpha) + \cos x(\sin 3\alpha - 3\sin\alpha) = 0$

$\boxed{4\cos^3\alpha} - \cancel{3\cos\alpha}$

$\sin x \cdot 4\cos^3\alpha = \cos x \cdot 4\sin^3\alpha$

$\frac{\sin x}{\cos x} = \frac{4\sin^3\alpha}{4\cos^3\alpha} \Rightarrow \tan x = \tan^3\alpha$

What is the value of

$\underline{\sin A.\cos A}.\tan A + \underline{\sin A.\cos A}.\cot A = ?$

(a) sinA

(b) cosA

(c) tanA

(d) 1

$$\sin A\cdot\cos A\left[\tan A + \cot\right] = \sin A\cdot\cos A\left[\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right]$$

$$= \cancel{\sin A\cdot\cos A}\left[\frac{\sin^2 A + \cos^2 A}{\cancel{\sin A\cdot\cos A}}\right] = \underline{1}$$

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